Jack of All Trades, Master of None: Multiracial Politicians and Political Representation*

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Abstract

How does racial competence shape Multiracial politicians' policy choices? Minority voters seek descriptive representation to achieve substantive outcomes. Multiracial politicians crystallize a tension in descriptive representation as their multiple perspectives and ambiguous identity complicate claims of racial competency. Using a formal model, I provide a pathway to understanding how electoral incentives condition Multiracial incumbents' choice to exaggerate policy or reveal competence. This clarifies tradeoffs multiracial officeholders face between appealing to multiple constituencies and signaling credibility to coracial voters. Extensions model how ideological polarization only benefits politicians when uncertainty about racial competency is high. Caucus membership provides insurance as it constrains policy choices and bolsters perceptions of competence. These findings advance theories of descriptive representation by showing that the multiplicity of perspectives and ambiguity embodied by Multiracial politicians complicates their ability to represent minority constituencies.

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> "I don't think you can consider the issue of mixed race outside of the issue of race. I think it's important to try to avoid that sense of exclusivity . . . Don't ever think of yourself as so unique that you divorce yourself from your communities."
>
> — Barack Obama (2005)

Introduction

What it means to represent minority interests in America remains a contested issue. Representation is not only about shared identity but about *racial competence*, or the ability to act effectively for minority constituents through shared experiences and contextual knowledge. By electing coracial politicians, voters reduce the information gap and place greater trust in their representatives. Yet whether descriptive identity is sufficient to ensure credible action remains unresolved, making racial competency the central tension in understanding how minority representation works.

Americans are increasingly identifying as mixed-race, leading to Multiracial politicians now comprising a growing minority of incumbents (Hardy-Fanta et al. 2013; U.S. Census Bureau 2022).[1] Multiracial politicians are framed as the "vaccine" to inter-racial conflict in the United States, in part because of their perceived ability to navigate shared experiences with multiple racial constituencies (Lemi 2021; Velasquez-Manoff 2017).[2] Politicians of mixed-race ancestry have likewise promoted the idea that their racial identity is strategic, enabling them to connect with multiple groups (Brown 2014; Lemi 2018).[3] As a result, the stakes of correctly assessing their racial competence are even higher. Multiracial politicians' strategies generate uncertainty about their competence, as constituents doubt their ability to act when situations demand contextual knowledge.

The interaction between voter perceptions and Multiracial politicians' attempts to show competence creates a dynamic of mutual influence. On one end, voters seek signals of racial competency to ensure their group-specific policy needs are addressed (Wamble 2025). Multiracial politicians, in turn, participate in identity-specific caucuses, engage in symbolic forms of representation, and support legislation benefiting minority interests as a way to demonstrate competence (Lemi 2018). Yet because voters cannot easily place Multiracial

1. However, see Starr and Pao (2024), who argues the 2020 spike reflects changes in measurement rather than a sudden demographic shift.

2. Pew (2015) found that one-in-five Multiracial Americans view themselves as a bridge between racial groups. Similarly, Hochschild and Weaver (2010) notes growing support for mixed-race heritage as a means of tearing down racial barriers.

3. I focus on White–Black Multiracial politicians, whose identities are especially salient given their shared racial history in the U.S. In particular, the legacy of hypodescent and the one-drop rule provides clear institutional and social incentives for Black-White Multiracials to align with monoracial Black interests (Davis 1991). The framework is tailored to this case but may also apply to other liminal groups; I return to the scope conditions in the discussion.

politicians as descriptive representatives, they often prefer in-group monoracial candidates instead (Lemi 2021).[4] This dynamic makes perceptions and actions interdependent: Multiracial politicians act to shape voter perceptions, while voter perceptions simultaneously condition which actions are seen as credible displays of racial competence.

However, the difficulty in this interplay is that it masks a principal-agent problem. In many constituencies, coracial alignment appears clear - because a voter shares observable characteristics with their representative, their interests are assumed to be aligned. Yet, as Wamble (2025) and Dovi (2002) emphasize, shared identity does not necessarily translate into racial competency. Competence is unobservable to constituents and can only be inferred through a politician's actions. This creates space for distortion, as politicians may alter their behavior to shape perceptions of their racial competency.

When a politician has shared experiences - or is racially competent - they can credibly navigate both universal and particularized issue domains. In contrast, when a politician lacks shared experience and voters require contextual knowledge to inform their policy decisions, the incumbent is unable to act effectively. Multiracial politicians offer a unique opportunity to examine this dynamic, as their ambiguous identity raises questions about their commitment to group-specific policy goals. This dynamic creates the core tension of my model: voters want a politician who can deliver when the issue of the day demands it, but are uncertain whether the incumbent can do so; incumbents want to be perceived as competent but face uncertainty about their ability. As a result, incumbents have incentives to distort policy to appear racially competent. More broadly, unpacking this interplay is essential for understanding how politicians' behavior varies across constituencies and settings with diverse policy needs.

My model provides a pathway to understanding how Multiracial politicians navigate perceptions of ambiguity and racial competency in an increasingly diverse electorate. I introduce contextual knowledge as a probability over the policy domain, which can either be universal or particularized. The incumbent officeholder faces uncertainty about the need for racial competency in the next period. Importantly, racial competency is conditional on the policy domain. Voters are likewise uncertain about the incumbent's ability to act with racial competence.[5] Each period, the incumbent selects a policy based on their type and domain, and voters decide whether to re-elect or replace them. Three main takeaways from the model are evident.

4. Lemi (2021) also shows that respondents are more supportive of mixed-race politicians when they have a shared identity than those who do not, highlighting the fact that any descriptive representation is better than none. This finding also provides evidence for the role shared experiences play in voters' evaluations of Multiracial politicians.

5. The game is structured such that the incumbent is an interracial politician. As such, competency here is less about a politician's general ability to handle a crisis (Bils 2023), but is a measure of racial competency or shared experiences. The asymmetric information environment aligns with the idea that voters are also unaware of the incumbent's in-group status. Because Multiracial politicians intentionally withhold their identity (Brown 2014; Lemi 2018), voters have to acquire information about their level of racial competency through their policy decisions.

First, I qualify the notion that the ambiguous identity of multiracial incumbents is a strategic advantage. While the baseline analyses indicate that when electoral incentives are strong, Multiracial politicians can exaggerate policy to secure re-election, they always risk being "outed" as incompetent, particularly if the future demands contextual knowledge and voters' prior beliefs are that the incumbent is racially competent. Moreover, voter welfare analyses highlight that the electorate is not better off from having Multiracial politicians ex-ante: Racial competency still matters.

Ideology serves as an important substitute for racial competency. When polarization is symmetric, Multiracial incumbents who lack racial competency can increase their bias and improve their re-election chances by moving the threshold at which they distort policy. Yet, their interaction effect limits the winning probability for the Multiracial politician. Polarization only benefits the incumbent when uncertainty about the need for racial competency in the next period is high; in universal domains, polarization diminishes the winning probability because exaggerating policy is no longer marginally beneficial. Significantly, this finding generalizes to Multiracial Americans at large, providing a mechanism for when they are likely to select ideologically extreme policies.

Finally, caucus membership and the constraints imposed serve as an important conduit of information for both the incumbent and the voter. Multiracial incumbents are keen on participating in identity-based organizations within the legislature (Lemi 2018). Part of the reason, I argue, is that constraining yourself via caucus membership serves as insurance against future periods where racial competency is required. Moreover, caucus membership provides opportunities for the incumbent to convey contextual knowledge to their constituency via leadership positions and symbolic representation (Canon 1999). Caucus constraints also serve as an important social monitor, forcing the Multiracial incumbent back in line with coracial policy goals (Rendleman 2023). This mechanism relates more broadly to racialized social constraints and Multiracial behavior at the mass level (White and Laird 2020).

This paper contributes to the study of Multiracialism by formalizing how racial ambiguity among Multiracial politicians generates uncertainty about racial competency. Existing theories of Multiracial politicians examine voter evaluations of mixed-race candidates (Lemi 2021; Leslie et al. 2022).[6] The core aspect of this literature is how voters perceive Multiracial politicians. Race serves as a heuristic for how well a politician can represent minority voters.[7] Multiracial politicians complicate the racial status quo, bridging inter-racial coalitions while alienating coracial constituencies (Lemi 2021; Leslie et al. 2022). Conversely, Lemi (2018) discusses the incentives Multiracial politicians face in choosing how to identify to their constituency. Multiracial politicians are acutely aware of how voters perceive them: their credibility depends both on their bridge-building appeal and on how convincingly they demonstrate racial competency to their community (Lemi 2021). Multiracial politicians

6. Mixed-race Americans face many mechanisms that drive identification (Rockquemore and Brunsma 2002). One mechanism, reflected appraisals, is the idea that Multiracial identity is driven by how others perceive them (Khanna 2004, 2010; Sims 2016). These perceptions are fluid and influenced by the actions taken by mixed-race Americans (Sims 2016).

7. Race signals perceived co-racial alignment, particularly in low-information elections (Dawson 1994; McConnaughy et al. 2010).

often present racial ambiguity as an asset, believing it allows them to appeal across constituencies (Brown 2014). Yet neither strand formalizes how voters' perceptions of racial competency structure the actions of multiracial politicians. My model directly addresses this gap by showing how electoral incentives condition when Multiracial incumbents choose to exaggerate policy or reveal competence, clarifying the strategic tradeoffs they face between appealing broadly as a coalition builder and signaling credibility to coracial voters.

Multiracial politicians crystallize a core tension in descriptive representation: the very multiplicity of perspectives they embody both expands and complicates claims of racial competence. My model addresses whether descriptive representation translates into substantive benefits for minority voters. Extant work discusses how descriptive representation leads to substantive outcomes for minority constituents because voters look for signals of racial competency (Dovi 2002; Mansbridge 1999; Wamble 2025; Williams 1993).[8] Young (1997), however, challenges this by noting that racial competency is not guaranteed by a singular group identity in the descriptive sense but is instead conditioned by one's social positioning. No single identity or actor can represent all interests; instead, social positioning opens the possibility for multiple actors to engage in the same perspective (Young 1997). On one hand, Multiracial representatives embody Young's proposition by condensing multiple social positions into a single actor, thereby potentially offering a plurality of perspectives at once. On the other hand, my model suggests that this multiplicity complicates the very logic of descriptive representation, as Multiracial politicians struggle to credibly demonstrate racial competence to their constituents. Even when Multiracial politicians are re-elected, they may have distorted policy through purely symbolic means. And while I do not debate the value of symbolic representation (Tate 2004), if the framing of Multiracial politicians is that they can bridge racial divides by meeting the needs of multiple constituencies, my analyses depict a more complicated reality.

My model extends existing work in the political economy literature (Bils 2023; Canes-Wrone, Herron, and Shotts 2001; Fox and Stephenson 2011; Maskin and Tirole 2004) by treating descriptive representation as an agency problem. Unlike prior accounts, which assume competence is exogenous (Bils 2023; Canes-Wrone and Shotts 2007), I demonstrate how perceptions of competence are shaped by ambiguity surrounding Multiracial politicians' identities. This ambiguity conditions both voters' assessments of credibility and politicians' strategic behavior in office. My model departs from this by making quality *endogenous to the policy domain*, demonstrating how racial competency influences whether minority voters perceive politicians as racially competent. Situating racial competency at the center of political economy models highlights how minority communities actively condition representatives' behavior.

In this respect, the underlying mechanism of my model is most closely related to Bils (2023), which also introduces competency as a guiding force in incumbents' decision-making.

8. while some scholars have argued that descriptive representation is neither necessary nor sufficient for substantive policy outcomes (Pitkin 1967; Swain 1995), others have demonstrated the clear implications of having coracial voices in Washington (Butler and Broockman 2011; Grose 2011; Haynie 2001; Hero and Tolbert 1995; Minta and Brown 2014; Preuhs 2006; Whitby 2000).

There are two mechanisms by which incumbents distort policy: anti-herding (or overreacting) (Levy 2004; Prendergast and Stole 1996), and posturing (or exaggerating) (Fox and Stephenson 2011). Incumbents exaggerate policy in an attempt to appear competent when they lack the necessary information to make informed decisions. Yet, what is left unclear from Bils (2023) is how shared experiences condition competency. For minority voters, navigating a crisis requires racial competency; yet, this community-specific dimension is left unaddressed (Dovi 2002; Mansbridge 1999; Williams 1993). My framework provides an avenue for incorporating this idea, situating racial competency at the center of accountability models for minority constituencies.

The rest of the article is structured as follows. First, I introduce the preliminaries of the model, discussing the players, actions, timing, information, and equilibrium concept. Next, I present the results of the baseline version of the model, highlighting the role that information plays in structuring behavior. In extensions, I introduce components significant to how we think about voter welfare, including information, legislative constraints, and polarization. Finally, I conclude with a synopsis of the findings and implications for empirical work.

Model Description

Players. There are three players in the model: an incumbent (I) who is seeking re-election, a challenger (C), and a voter (V).

Timing. The model has two periods, $t \in \{1, 2\}$. In each period, an incumbent chooses a policy $x_t \in X = \mathbb{R}$. At the end of the first period, a voter observes x_t and chooses to re-elect the incumbent or elect the challenger. I exclude legislative caucus constraints from the baseline model to isolate the core role that racial competency plays in influencing policy distortion.

Preferences. In the baseline model, players have the same policy preferences, represented by an ideal point ω_t. The voter's utility is quadratic-loss and represented by $-(x_t - \omega_t)^2$. The incumbent's utility is

$$-(x_t - \omega_t)^2 + \mathbb{1}_t \beta,$$

where $\beta \in (0, \infty)$ is the office benefit a politician receives for each period they hold office, and $\mathbb{1}_t$ indicates whether the politician is in office. Politicians aim to maximize their utility over the two periods. In extensions, ideology is included as an additive parameter in the incumbent's utility to understand how racial competency interacts with polarization.

Information. In each period, nature randomly determines the state of the world, ω_t. The state of the world represents the ideal policy position and is unknown to voters and uninformed incumbents. Let ω_t be drawn from a known distribution F, which has a probability density f. Furthermore, let F have full support over $\mathbb{R}$ and be distributed with a mean of 0 and $\sigma^2 > 0$. Finally, I assume that ω_1 and ω_2 are uncorrelated.

A politician's competence is conditional on the policy domain. I assume that an office-holder is aware of the domain of the current period when making policy choices; however, there is uncertainty about the future domain. I introduce a new parameter, ρ, which captures the probability distribution over policy domains. Specifically, the policy domain d_t is drawn by nature with probability $\rho = \Pr(d_t = U)$ for the universal domain (U), and $1 - \rho = \Pr(d_t = P)$ for the particular domain (P). This addition highlights that electoral contexts are not homogeneous. The environment in which a politician operates may emphasize either broad, universal concerns or group-specific, particularized concerns.

Whether a politician is viewed as racially competent depends on the policy domain. Let a politicians' type be defined as $\theta_j \in \{H, L\}$ for $j \in \{I, C\}$. Informed types (H) are fully aware of ω_t and are competent across both domains. Uninformed types (L) are competent only in the universal domain; when placed in the particular domain, they are unable to internalize ω_t. Uncertainty about type θ_j is represented by a common prior that a politician is racially competent with probability $\pi_j \in (0, 1)$.

The core contribution is that racial competency is no longer viewed as an inherent trait (Bils 2023), but rather as something that depends on the policy environment. This captures how minority representation is contingent not just on whether an official is "competent" in the abstract, but on whether their competence aligns with the domain-specific demands of their constituency. Within the baseline and in extensions, racial competence plays a significant role through information, impacting not only the incumbent's behavior but also voter welfare.

Equilibrium. I study perfect Bayesian equilibria (PBE). Specifically, let $\mu : X \to [0, 1]$ represent a mixed strategy for the voter, where $\mu(x)$ is the probability of re-electing the incumbent following their policy choice x. Strategies are sequentially rational given beliefs, and beliefs are derived from Bayes' Rule whenever possible. For the politician, a mixed strategy is defined by $\sigma_t : \mathbb{R} \cup \{\phi\} \to \Delta(X)$, where σ_t maps their private information (informed $\mathbb{R}$ or uninformed ϕ) to a probability distribution over X. To restrict off-path beliefs, I focus on PBE satisfying D1 (Cho and Kreps 1987). Intuitively, upon observing an off-path policy, the voters' belief would assign a higher probability to the incumbent type that would have more to gain from deviating.

Results

The results section is organized as follows. In the baseline model, I analyze how uncertainty about an officeholder's racial competence shapes equilibrium policymaking and electoral outcomes. For voters, this is apparent through their decision rule. For incumbents, racial competency affects both the extremity and willingness to distort policy, improving their probability of re-election. Voter welfare analysis shows that perceptions are the primary driver of an officeholder's willingness to distort policy. In extensions, racial competency diminishes the effectiveness of polarization on the incumbent's re-election chances. At the same time, uncertainty about racial competency shows how caucus membership acts as a safeguard against the need for contextual knowledge in future periods.

Baseline Findings

In the second period, there is no electoral incentive, so the officeholder selects their optimal policy based on their information about ω_2. The voter's continuation value from electing the challenger depends on three main components: The probability that the challenger is uninformed $(1-\pi_C)$, the probability that the current period requires contextual knowledge $(1-\rho)$, and the policy loss from the low-type selecting $x_1 = 0$ $(-\sigma^2)$. Formally, the voter's continuation value from re-electing the incumbent is $V(I \mid x_1) = -[1-\tilde{\pi}_I](1-\rho)\sigma^2$, while the value of electing the challenger is, $V(C) = -[1-\pi_C](1-\rho)\sigma^2$.

Reducing the need for racial competency raises the voter's continuation value, as higher values of ρ increase the expected payoff from electing the challenger. Lemma 1 below establishes this result.

Lemma 1 (Impact of ρ on Continuation Values)**.** *The voter's continuation value from electing the challenger is strictly increasing in ρ, and converges to* 0 *as ρ approaches* 1*.*

Lemma 1 shows that the policy domain directly conditions the voter's continuation value by reducing the risk of policy loss from an uninformed challenger. As ρ approaches 1, the need for racial competency decreases, and the voter's decision rule is determined entirely by the incumbent's first-period behavior. Conversely, as ρ approaches 0, the need for racial competency increases, and the voter compares the incumbent's posterior probability of being competent to the challenger's prior. An analogous condition applies to the incumbent's continuation value.[9] Lemma 1 also impacts the voter's decision rule by collapsing the continuation values from both the incumbent and challenger to 0 as ρ approaches 1. Therefore, the posterior belief about the incumbent is only significant to the voter's calculus when $\rho < 1$.

Next, to define the voter's decision rule, sequential rationality requires that the voter select the candidate that provides the greatest continuation value, as there is no commitment

9. All proofs are provided in the Appendix.

mechanism. Let $\tilde{\pi}_I$ be the voter's posterior belief that the incumbent is competent. Then the voter's decision rule is determined by the posterior belief that the incumbent is racially competent and the prior belief about the challenger.

Lemma 2 (Voter Decision Rule)**.** *After observing first-period policy x_1 in equilibrium, the voter (1) reelects the incumbent if $\tilde{\pi}(x_1) > \pi_C$, (2) reelects the challenger if $\tilde{\pi}(x_1) < \pi_C$, and (3) is indifferent if $\tilde{\pi}(x_1) = \pi_C$.*

In equilibrium, Lemma 2 shows that the condition on beliefs is pinned down by a comparison of continuation values. If $V(I \mid x_1) > V(C)$, the voter will re-elect the incumbent. Similarly, if $V(I \mid x_1) < V(C)$, they will elect the challenger. When $V(I \mid x_1) = V(C)$, the voter mixes and re-elects the incumbent with some probability $\mu(x_1) \in [0,1]$. Importantly, racial competency enters into the voter's decision rule through their continuation values. As such, racial competency directly affects voters' posterior and prior beliefs. This will become useful in an extension where voter welfare is examined.

Complete Information Benchmark

I now characterize first-period behavior under complete information. This baseline illustrates electoral incentives when the voter's uncertainty about type is removed. Voters are fully aware of the incumbent's type, so the election is a foregone conclusion. Racial competency structures the incumbent's optimal action as follows: When $d_1 = U$, both types select $x_1 = \omega_1$; conversely, when $d_1 = P$, the competent type matches the state while the incompetent incumbent chooses $x_1 = 0$. This is analogous to Bils (2023) with the critical departure being that the second-period policy setting is unknown, creating additional uncertainty for the low-type. Remark 1 summarizes the equilibrium behavior.

Remark 1. *Under the complete information benchmark, there is a subgame perfect Nash equilibrium in which the high-type incumbent always selects $x_1 = \omega_1$, and the low-type incumbent selects*

$$x_1 = \begin{cases} \omega_1, & \text{if } d_1 = U, \\ 0, & \text{if } d_1 = P. \end{cases}$$

The voter always reelects the high-type and rejects the low type.

Remark 1 helps ground the role of racial competency in both the voter's perceptions and the incumbent's actions. Here, voters know the competent incumbent; as a result, the uninformed type has no incentive to distort policy to be perceived as competent. However, when voters cannot observe the incumbent's type, politicians strategically exaggerate policy when office rents are high. The degree of distortion is structured by racial competency.

Uncertainty about Racial Competency

I now introduce voter uncertainty about the incumbent's racial competency. Under incomplete information, incompetent types may be willing *posture* and select policy positions in equilibrium that force the informed incumbent to *overreact* to signal to voters their competency. Specifically, following Bils (2023),

Definition 1 (Overreacting and Posturing)**.** *An informed incumbent overreacts when $\omega > 0$ and they choose $x > \omega$, or when $\omega < 0$ and they choose $x < \omega$ (Levy 2004; Prendergast and Stole 1996). An uninformed incumbent postures when they choose any $x \neq \mathbb{E}[\omega] = 0$ (Fox and Stephenson 2011).*

I assume that the incumbent is aware of the policy domain in the first period.[10] Because racial competency is not required when $d_1 = U$, both incumbent types match the state. As a result, I examine first-period behavior when $d_1 = P$ unless noted otherwise. To introduce the dynamics of this section, note that the high-type never deviates from equilibrium behavior, as they know the state in each period. Therefore, the incentive for the uninformed incumbent to deviate guides first and second-best behavior. Let the incentive compatibility constraint for the uninformed type be

$$\underbrace{\beta - \sigma^2}_{\text{Equilibrium Payoff}} \quad \underbrace{-[1 - \pi_C]\sigma^2}_{V(C)} \quad = \quad \underbrace{2\beta - (2 - \rho)\sigma^2}_{\text{Total Expected Payoff}} \, . \tag{1}$$

where the equilibrium payoff is the uninformed type's expected utility from choosing $x_1 = 0$ and getting kicked out of office. The total expected payoff is if the uninformed type postured and selected a different policy, resulting in their re-election. This is analogous to Bils (2023) except for the uncertainty ρ introduces in the second period. Solving for this provides the office rent cut-points that delineate two cases: low-value office, when $\beta \leq \pi_C \sigma^2 - \rho\sigma^2$ and high-value office, where $\beta > \pi_C \sigma^2 - \rho\sigma^2$.

Low-value Office

When office rents are low, uninformed incumbents face no incentive to posture as risking the policy loss from deviating is too great. Therefore, they select their optimal first-best outcome, $x_1 = 0$, and are kicked out of office. Remark 2 highlights this finding, which is standard in accountability models with competence considerations (Bils 2023; Canes-Wrone, Herron, and Shotts 2001).

10. It is safe to assume that politicians are aware of their constituents' present policy needs by performing constituency service, yet still unaware of what the future holds (Fenno 1978). For example, while a minority community may not require particularized expertise in the current period, a racialized event may trigger the need for it tomorrow. Future research could expand the setting to a model where there are also particularized interest groups demanding racial competency from elected officials.

Remark 2. *If office rents are low so that $\beta \leq \pi_C\sigma^2 - \rho\sigma^2$, then there exists a perfect Bayesian equilibrium in which the high-type incumbent selects $x_1 = \omega_1$ and the low-type incumbent selects $x_1 = 0$.*

Under this condition, types fully separate in equilibrium; voters update their beliefs with probability 1 that the incumbent is racially competent, ensuring they re-elect the informed type and reject the uninformed type. Substantively, when office benefits are low, an uninformed incumbent prefers to accept defeat rather than posture to distort beliefs about their commitment to delivering on particularistic policy.

High-value Office

Suppose now that the office benefits are high ($\beta > \pi_C\sigma^2 - \rho\sigma^2$). In this case, the uninformed type has an incentive to *posture in order to appear racially competent.* To characterize this tradeoff, assume there exists some policy choice $-x^2$ that leaves the uninformed type indifferent between following the equilibrium strategy and deviating. Following Bils (2023), the incentive compatibility condition is

$$\underbrace{\beta - \sigma^2}_{\text{Equilibrium Payoff}} \quad \underbrace{-[1-\pi_C]\sigma^2}_{V(C)} \quad = \quad \underbrace{-x^2}_{\text{Policy Indifference}} \quad + \underbrace{2\beta - (2-\rho)\sigma^2}_{\text{Total Expected Payoff}} . \tag{2}$$

Solving yields the policy bounds $\underline{x}^P(\rho)$ and $\overline{x}^P(\rho)$ at which the uninformed incumbent is indifferent: $\overline{x}^P(\rho) = \omega^P(\rho) = \sqrt{\beta - \pi_C\sigma^2 + \rho\sigma^2}$. At this point, the model follows Bils (2023) in using incentive compatibility to locate the thresholds that discipline behavior. The critical departure, however, lies in the uncertainty surrounding the second period. Uncertainty about the need for racial competency affects the incumbent's willingness and the extent of policy extremity. In earlier models, the environment is fixed, so distortion depends only on office rents and type uncertainty. Here, the possibility that voters may require contextualized racial competence in the next period directly enters the cut-point through the $\rho\sigma^2$ term. This modification shifts both *how far* the uninformed type will posture and *how much* the informed type must exaggerate to signal competence.

The probability that ω lies between 0 and $\omega^P(\rho)$ determines the incumbent's willingness to distort policy (equivalently, the lower bound lies between $-\omega^P(\rho)$ and 0). At the boundary $x_1 = \overline{x}^P(\rho)$, the voter is indifferent and by Lemma 2, re-elects the incumbent with some probability $\mu(x_1) \in [0, 1]$. Accordingly, the cut-points defining the probability that ω_1 lies within the threshold range are

$$\overline{\Sigma} = \frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}\big(F(\omega^P(\rho)) - F(0)\big), \quad \underline{\Sigma} = \frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}\big(F(0) - F(-\omega^P(\rho))\big).$$

Here, racial competency enters through the state threshold, amplifying the mixing probability for the uninformed incumbent as well as their policy extremity. With that in mind, I characterize the set of Perfect Bayesian Equilibria of the model.

Proposition 1 (Perfect Bayesian Equilibria with High Office Rents)**.** *The set of Perfect Bayesian Equilibria of the model is characterized as follows:*

1. ***Voter strategy.***
   - *If $x_1 \in (\underline{x}^P(\rho),\ \overline{x}^P(\rho))$, the voter rejects the incumbent.*
   - *If $x_1 \leq \underline{x}^P(\rho)$ or $x_1 \geq \overline{x}^P(\rho)$, the voter re-elects the incumbent.*
2. ***Informed incumbent.***
   - *If $\omega_1 \in [0, \omega^P(\rho))$, the informed type overreacts by setting $x_1 = \overline{x}^P(\rho)$.*
   - *If $\omega_1 \in (-\omega^P(\rho), 0]$, the informed type overreacts by setting $x_1 = \underline{x}^P$.*
   - *If $|\omega_1| > \omega^P(\rho)$, the informed type plays the first-best by choosing $x_1 = \omega_1$.*
3. ***Uninformed incumbent.***
   - *With probability $\overline{\sigma} \in [0, \min\{1, \overline{\Sigma}\}]$, the uninformed type postures right by choosing $x_1 = \overline{x}^P(\rho)$.*
   - *With probability $\underline{\sigma} \in [0, \min\{1-\overline{\sigma}, \underline{\Sigma}\}]$, the uninformed type postures left by choosing $x_1 = \underline{x}^P(\rho)$.*
   - *With residual probability $1 - \underline{\sigma} - \overline{\sigma}$, the uninformed type plays the first-best by setting $x_1 = 0$.*

Off the path of play, assume the voter updates their belief with probability 1 that the incumbent is uninformed.

Under Proposition 1, the incumbent distorts policy in order to signal racial competency. If $x_1 \in (\underline{x}^{P(\rho)},\ \overline{x}^{P(\rho)})$, the voter infers that the incumbent is uninformed and turns to the challenger. By contrast, when policy is set at or beyond the thresholds $|x_1| > \overline{x}^P(\rho)$, the voter updates with probability 1 that only an informed type would choose such a position, and re-elects the incumbent. In this region, the uninformed type can mix with probability $\Sigma \in [\underline{\Sigma}, \overline{\Sigma}]$. As long as the uninformed type does not mimic too often, the voter remains willing to retain the incumbent. An analogous result is evident for $x_1 = \underline{x}^P(\rho)$.

This structure parallels Bils (2023), in which informed incumbents exaggerate to distinguish themselves, and uninformed incumbents mix to exploit this exaggeration. However, because an incumbent's behavior is structured not only by their type but by the need for contextual knowledge, my model expands the possible actions an incumbent can take to distort policy. In that respect, Bils (2023) becomes a special case where $\rho = 0$. Here, uncertainty alters the incentives on both sides, expanding the range in which uninformed types can posture and raising the level at which informed types must exaggerate to be believed. For the informed type, behavior depends on the state. When the state is extreme relative to the threshold, the informed type chooses their first-best policy. But when the state is more moderate and close to zero, they cannot simply match ω, since voters reject any policy choice inside the interval $(\underline{x}^P(\rho),\ \overline{x}^P(\rho))$. To demonstrate competency, the informed type exaggerates, selecting a boundary policy such as $x_1 = \overline{x}^P(\rho)$. The uninformed type mixes between posturing at the cutoffs, $x_1 = \overline{x}^P(\rho)$ or $x_1 = \underline{x}^P(\rho)$, and sometimes setting $x_1 = 0$. Because they cannot know the state ex ante, their survival depends on being seen as competent often enough to survive into the next period.

Proposition 1 connects directly to the broader challenge introduced at the start of the paper. Multiracial incumbents, whose descriptive ties are ambiguous, are especially pressured to distort policy in order to be perceived as racially competent. Because their ambiguity causes even competent incumbents' coracial status to be questioned, they feel the need to overreact and exaggerate policy to demonstrate racial competence. For uninformed incumbents, this same ambiguity creates opportunities to slip through the cracks and be perceived as racially competent despite lacking the ability to deliver particularistic policy. In this sense, racial ambiguity amplifies the incentives on both sides of the signaling game. It raises the cost of credible separation for the competent type, while expanding the scope for opportunism by the uninformed. The discussion section returns to this point, situating the model's logic within broader debates on race and representation.

To rule out other potential equilibria, I refine Proposition 1 using the set of non-off-the-path beliefs (Cho and Kreps 1987). The next remark demonstrates this refinement.

Remark 3. *The equilibria identified in Proposition 1 survive D1.*

Any policy choice $x \in \big(\underline{x}^P(\rho),\, \overline{x}^P(\rho)\big) \setminus \{0\})$ is off the path of play. Both types benefit from selecting $x \in (\underline{x}^P(\rho),\ \overline{x}^P(\rho))$. However, the unformed incumbent's utility from deviating are larger than the informed type. Therefore, D1 requires the voter to update that the incumbent is the uninformed type with probability 1. This finding is equivalent to Bils (2023).

Contextual knowledge, ρ, enters the uninformed type's calculus through their upper and lower policy bounds. As ρ increases, the incentive to posture at the cut-point grows, since voters are less focused on racial competence and more willing to accept exaggerated policies as credible signals. Proposition 2 characterizes this comparative static by showing how ρ shifts the extremity of equilibrium policy choices.

Proposition 2 (Impact of ρ on Policy Extremity)**.** *An increase in the probability that the second-period policy domain is universal leads to an increase in the extremity of the policy choice; that is,*

$$\frac{\partial \overline{x}^P(\rho)}{\partial \rho} > 0 \quad \textit{and} \quad \frac{\partial \underline{x}^P(\rho)}{\partial \rho} < 0.$$

Furthermore, as $\rho \to 1$,

$$\lim_{\rho \to 1} \overline{x}^P(\rho) = \sqrt{\beta - (\pi_C - 1)\sigma^2} \quad \textit{and} \quad \lim_{\rho \to 1} \underline{x}^P(\rho) = -\sqrt{\beta - (\pi_C - 1)\sigma^2}.$$

Proposition 2 shows that uncertainty about the need for contextual knowledge directly conditions the extremity of incumbents' actions. As the probability of a universal second period increases, the cut-points that discipline uninformed types shift outward, making more extreme policies credible. Figure 1 illustrates this comparative static: the dotted regions depict the expansion of the policy bounds as ρ rises. The red markers trace how the uninformed type's posturing cut-points, $\overline{x}^U(\rho)$ and $\underline{x}^U(\rho)$, move away from the origin. The braces emphasize that a higher ρ increases the distance between the inner and outer bounds, highlighting the growth in policy extremity. Substantively, this means that when voters are more confident that the future domain is universal, it becomes easier for an uninformed incumbent to posture as racially competent. Voters are more willing to re-elect them in equilibrium. Proposition 3 formalizes this result.

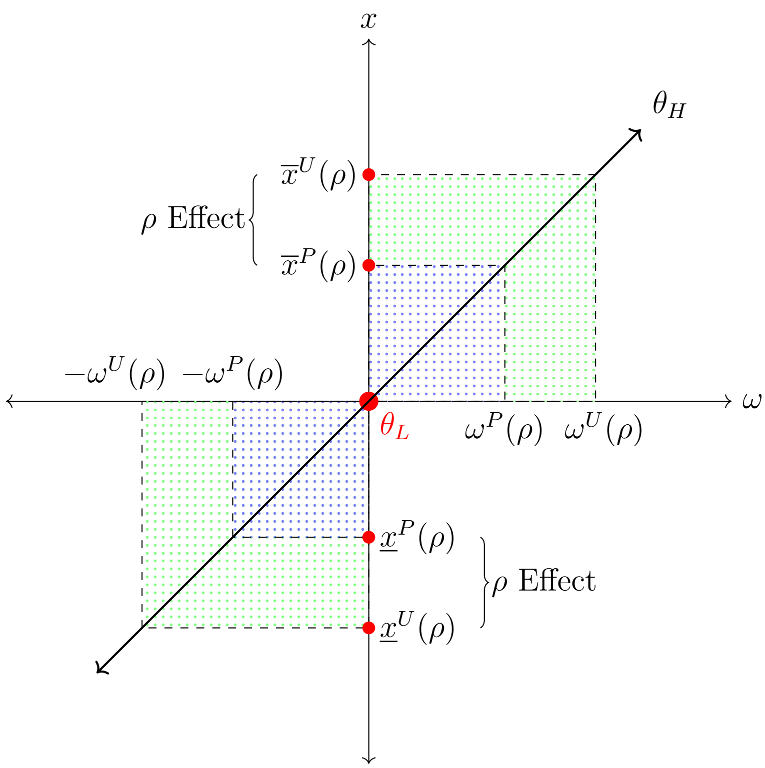

Figure 1: The black arrows represent policy choices by the high type. The red circles mark the policy cut-points that shift with ρ. $\overline{x}^P(\rho)$ and $\underline{x}^P(\rho)$ depict the boundaries of extremity, illustrating that higher ρ increases policy extremity.

Proposition 3 (Impact of ρ on the Winning Probability for the Uninformed Incumbent)**.** *As ρ increases, the winning probability for the uninformed incumbent increases.*

For Multiracial incumbents in particular, Propositions 2 and 3 demonstrate the constraints that contextual knowledge imposes on the actions chosen in equilibrium. When voters require racial competency in the next period, Multiracials condition their posturing on these perceptions, knowing that they are unable to meet the demands of their constituency. However, when racial competency is not required, Multiracial incumbents are more willing to set policy at a higher (lower) bound in order to be perceived as the competent type.

Voter Welfare

Next, I examine voter welfare and the implications of the need for contextual knowledge on outcomes. Remark 4 depicts the impact of prior beliefs on voter welfare.

Remark 4. *Voter welfare satisfies:*

1. *Increasing the prior belief that the challenger is informed always increases welfare:*

$$\frac{\partial W}{\partial \pi_C} > 0.$$

2. *Increasing the prior belief that the incumbent is informed decreases welfare whenever*

$$\beta > (2-\rho)\sigma^2.$$

Voter welfare increases in challenger quality and decreases in incumbent quality when rents from office are large (Bils 2023). This parallels previous results, but the addition of the ρ parameter modifies the conditions under which it applies: uncertainty about the need for contextual knowledge shifts the thresholds at which rents reduce welfare.

Bils (2023) engages with two implications of voter welfare on incumbent policymaking authority: when office benefits are low, policymaking should be made by the incumbent; otherwise, either the incumbent should be stripped of policymaking authority or the voter must always re-elect the incumbent. The logic is that when re-election concerns are muted, incumbents are less inclined to distort policy. I qualify both implications by showing that the ρ parameter alters this tradeoff. Specifically, Implication1 demonstrates that uncertainty about the need for racial competence both circumscribes the voter's selection mechanism and lowers the rent threshold at which incumbents posture.

Implication 1. *Increasing the probability that the next period is universal lowers the β required for π_I to reduce voter welfare.*

As ρ increases, the office benefit threshold decreases, raising the incentive for distortion and worsening voter welfare. In effect, contextual knowledge renders the rent cutoff endogenous to the policy domain. When the probability of a universal policy is high, incumbents are more willing to posture, and voters are less able to rely solely on rents to discipline behavior. Racial competency, therefore, functions as a selection mechanism, but one whose efficacy depends on the interaction of ρ and β.

Implication 1 highlights that the screening role of elections depends jointly on ρ and β. What remains is to consider how voter beliefs about incumbent competence interact with this tradeoff. Implication 2 turns to this issue, showing how changes in π_I alter welfare once contextual knowledge is taken into account.

Implication 2. *If office benefits are low, elections screen for racially competent incumbents and improve voter welfare. When office benefits are high, incumbents posture to appear competent, reducing welfare unless policymaking authority is constrained or contextual knowledge is more strictly required.*

This implication parallels Mansbridge (2009) argument that descriptive representation is best seen as a selection mechanism. Voters choose representatives not only for racial competence but also for their capacity to translate that competency into substantive policy. When office incentives are low, descriptive cues and the accountability mechanism improve voter welfare; however, when incentives are strong, incumbents are more willing to distort policy, which undermines this mechanism, resulting in elections being less effective as a tool for racial screening. By lowering the office benefit threshold, Implication 1 shows that higher ρ makes elections more fragile as a selection device.

Voter welfare is shaped not only by policy outcomes but also by prior beliefs about the racial competency of incumbents and challengers. For Multiracial incumbents, racial ambiguity makes these priors especially salient. If voters view the incumbent as a coracial, they are more likely to interpret their actions as evidence of competence. In contrast, if they view the challenger as more racially competent, the incumbent faces a narrower scope for posturing. In this way, perceptions of racial competency act as a channel through which descriptive representation influences strategic behavior.

From the incumbent's perspective, these perceptions determine the attractiveness of mixing. A higher belief in the incumbent's competence increases the credibility of their postures and makes mixing more valuable; conversely, a higher belief in the challenger's competence reduces the payoff to mimicry and constrains the uninformed type's strategic options. Proposition 4 formalizes this relationship.

Proposition 4 (Impact of Prior Beliefs on Mixing Probabilities)**.** *Let the mixing probabilities for the uninformed incumbent be $\Sigma \in (\underline{\Sigma}, \overline{\Sigma})$. Then,*

1. *An increase in the prior belief that the incumbent is high-type raises the probability of posturing right, $\frac{d\overline{\Sigma}}{d\pi_I} > 0$, and lowers the probability of posturing left, $\frac{d\underline{\Sigma}}{d\pi_I} < 0$.*
2. *An increase in the prior belief that the challenger is high-type lowers the probability of posturing right, $\frac{d\overline{\Sigma}}{d\pi_C} < 0$, and raises the probability of posturing left, $\frac{d\underline{\Sigma}}{d\pi_C} > 0$.*

By connecting voter beliefs to the incumbent's actions, Proposition 4 illustrates how racial competency conditions both the direction and magnitude of mixing. The direct effect is that when voters are more confident the incumbent is competent ex-ante, they are more likely to view them as racially aligned, which increases the credibility of posturing. The indirect effect follows from the same belief. Stronger perceptions of racial competency raise the pressure on the incumbent to exaggerate in equilibrium, enlarging the scope of mixing. By contrast, when voters believe the challenger is racially competent, the uninformed incumbent has less room to maneuver. Posturing carries a higher risk of exposure, and the safer strategy is to minimize losses by selecting the first-best outcome. The magnitude of this tradeoff is further shaped by contextual knowledge: as the probability of a universal domain rises, the thresholds that discipline posturing shift, making the incumbent's willingness to mix less sensitive to perceptions of racial competence.

Substantively, this connects to the literature on reflected appraisals among Multiracial Americans. Identification is shaped less by self-perception than by how Monoracial peers evaluate mixed-race individuals (Khanna 2004, 2010; Sims 2016), and these evaluations are moderated by racial context (Rockquemore and Brunsma 2002). The model captures this dynamic by treating voter beliefs about competence as priors that directly affect the incumbent's willingness to mix. For Multiracial incumbents, being perceived as coracial heightens the pressure to act within group constraints, especially when racial competence is demanded. In equilibrium, this pressure manifests as a greater incentive to posture.

Extensions

While the baseline model provides a foundation for the role racial competency plays in structuring Multiracial behavior, there are further political considerations by which racial competency affects incumbents' behavior. Voter welfare is structured by perceptions of the incumbent's competency, influencing their willingness to posture. Ideology serves as a substitute for racial competency, but uncertainty impacts the overall value of polarization. Finally, legislative constraints through caucus membership serve as insurance for Multiracial politicians, safeguarding against the uncertainty of future demands for racial competency.

Ideology

I extend the game to a setting where politicians have some ideological bias, L or R. While the previous analyses provide clear insights into how contextual knowledge structures incumbents' incentives, polarization is an important factor to consider when thinking about voter perceptions and racial competency. Incumbents who lack racial competency may polarize their policy position, assuming that particularized needs can be met with ideological extremity. As in Bils (2023), to introduce polarization, assume that the incumbent has some bias R and the challenger a bias L, with $L < 0 < R$. The updated payoff for the incumbent is $-(x-\omega_t-R)^2$, while the voter's payoff remains $-(x-\omega_t)^2$. The reduced-form expressions for the upper and lower bounds of bias R, and the state threshold are

$$\overline{R} = \sqrt{L^2 + (1-\pi)(1-\rho)\sigma^2},$$

$$\underline{R} = \sqrt{\max\{0, L^2 - \left(\rho + \pi - \pi\rho\right)\sigma^2},$$

$$\omega_R^P = \sqrt{\beta + \rho\sigma^2 - \pi\sigma^2 + (R-L)^2}.$$

Furthermore, I normalize the prior beliefs about the incumbent and challenger's competency such that $\pi_I = \pi_C = \pi$. Polarization is defined as the difference between L and R.

I begin by briefly analyzing lopsided elections. To define lopsided elections, assume that the voter has some preference for the incumbent who is closer to their ideological position. Then, for some $R < \underline{R}$, the voter will re-elect the incumbent even if they are uninformed; conversely, if $R > \overline{R}$, the challenger always wins even if the incumbent is racially competent.

In lopsided elections, both sets of politicians select their first-best option, given that the election is a foregone conclusion. What matters here is that ρ factors into the upper and lower bounds of the incumbent's ideological extremity. As ρ increases, the upper bound decreases while the lower bound approaches 0. Otherwise put, the incumbent has to polarize less to maintain their lopsided victory when racial competence is not required. This has important implications for competitive elections, where both the willingness and threshold to distort are central.

Unlike a lopsided election, competitive settings introduce the incentive to distort policy again. Let $\underline{R} < R < \overline{R}$ define a competitive election. Furthermore, let $\overline{x}_R^P = R + \omega_R^P$ and $\underline{x}_R^P = R - \omega_R^P$. I define the mixing probability for the uninformed type as:

$$\overline{\Sigma}_R = \frac{\pi}{1-\pi} \frac{1 - \pi - \frac{(R^2 - (R^2))}{\sigma^2}}{\pi + \frac{(R^2 - (R^2))}{\sigma^2}} \big(F(\omega_R^P) - F(0)\big),$$
$$\underline{\Sigma}_R = \frac{\pi}{1-\pi} \frac{1 - \pi - \frac{(R^2 - (R^2))}{\sigma^2}}{\pi + \frac{(R^2 - (R^2))}{\sigma^2}} \big(F(0) - F(-\omega_R^P)\big).$$

Here, racial competency operates directly through the condition for what makes an election competitive, but also indirectly through the state threshold. Moreover, racial competency directly affects the incumbents' threshold for policy extremity. Therefore, the need for contextual knowledge affects not only the winning probability for the incumbent but also how both polarization and contextual knowledge impact the extremity of policy distortion.

Lemma 3 defines the equilibrium for competitive elections.

Lemma 3 (Competitive Elections)**.** *Suppose the election is competitive. Then,*

1. *There is a perfect Bayesian equilibrium analogous to proposition 1.*
2. *As polarization increases, so does* ω_R^P*.*

When elections are competitive, voters are willing to re-elect the incumbent as long as $|x_1| \geq \overline{x}_R^P$. This creates an incentive for the incumbent to overreact or posture strategically in order to secure re-election. Importantly, the threshold for policy distortion, ω_R^P, rises with polarization. Here, racial competency operates by affecting both polarization and the threshold for policy distortion. As racial competence becomes less important tomorrow, the threshold for which the incumbent can exaggerate policy widens; yet, ρ also narrows the bounds at which the election is competitive, circumscribing the incumbent to a set of smaller R required to win.

As Bils (2023) notes, asymmetric polarization makes it difficult to characterize the incumbent's mixing behavior. Competitive elections can become lopsided when one politician diverges at a faster rate, making it unclear how racial competency is structuring behavior. To facilitate comparative statics, I therefore assume symmetric polarization by letting the incumbent and challenger's bias diverge at equal rates. This assumption clarifies the role of racial competency by isolating its effect on the incumbent's winning probability and the extremity of distortion.

Proposition 5 (Comparative Statics Under Symmetric Polarization)**.** *Suppose the incumbent's and challenger's biases are equidistant* ($R = -L$)*. Then,*

1. ***Effect of polarization.*** *The incumbent's winning probability increases in polarization:*
$$\frac{\partial P_{win}^{L}}{\partial R} > 0.$$
2. ***Effect of racial competency.*** *The incumbent's winning probability increases in the probability of a universal second period:*
$$\frac{\partial P_{win}^{L}}{\partial \rho} > 0.$$
3. ***Interaction effect on the state threshold.*** *The marginal benefit of polarization on the state threshold decreases as* ρ *increases:*
$$\frac{\partial \omega_{R}^{P}}{\partial R\, \partial \rho} < 0.$$
4. ***Interaction effect on the winning probability.*** *The marginal benefit of polarization on the incumbent's winning probability decreases as* ρ *increases:*
$$\frac{\partial P_{win}^{L}}{\partial R\, \partial \rho} < 0.$$

Proposition 5 establishes that under symmetric polarization, the winning probability for the uninformed type increases as polarization increases. Because polarizing the election increases the threshold for policy distortion, both types of incumbents are more willing to overreact or posture in equilibrium. When we examine the effect of needing contextual knowledge in the next period, a similar pattern emerges. Racial competency affects the incumbent's mixing probability by increasing their willingness to posture in equilibrium.

As the probability that the next period becomes universal increases, the incumbent faces a greater incentive to mix. Therefore, their winning probability also rises, as depicted in part two of Proposition 5.

Polarization and racial competency are substitutes in the electoral arena. If the Multiracial incumbent is unable to deliver effective, particularized policy in the next period, they can polarize and secure re-election. Similarly, a reduction in the salience of contextual knowledge has a similar effect by loosening the voter's standard for racial competence. This equivalence is summarized in the following corollary.

Corollary 1 (Electoral Equivalence)**.** *ρ and R are electorally equivalent under symmetric polarization.*

While they are electoral substitutes, the incumbent is unable to know ex ante which behavior to take to secure re-election. Uncertainty about contextual knowledge may drive the incumbent to polarize now to improve perceptions that they are racially competent; however, this may also potentially stifle them with voters, as they are less willing to tolerate polarizing elections when the next period is universal. The bounds of competitive elections shrink as ρ increases. Part 3 of Proposition 5 begins to unpack the joint effect of polarizing and the need for racial competency on the incumbent's policy distortion threshold. As the probability that ρ increases, polarizing the election leads to a constraint on the extremity of their policy. At first glance, this appears to contradict the earlier claim that universal periods lead to more posturing. The distinction, however, is that this highlights a marginal effect: for a one-unit increase in polarization, there is a diminishing effect on shifting the policy distortion threshold. Moreover, this connects to a theme seen throughout the ideological extension - voters require less polarization in universal periods. This phenomenon was found in lopsided and asymmetric competitive elections. Incumbents are better off following their first-best behavior rather than polarizing. But, because they want to be perceived as racially competent, they polarize to safeguard against the possibility that the second period requires contextual knowledge. This effect has important empirical implications. In Davenport (2016) analysis of Multiracial attitudes, she finds that on issues related to race, mixed-race respondents are more liberal than their Monoracial minority peers. Implication 3 provides a mechanism for this pattern.

Implication 3. *For Multiracial incumbents, when racial competency is required, uninformed types are more likely to polarize.*

Because racial competence affects not only the bounds of policy distortion but the mixing probability, the need for contextual knowledge both directly and indirectly affects the incumbent's winning probability. Part 4 of Proposition 5 shows that as polarization and the probability of a universal period rise together, the incumbent's advantage shrinks. Again, the same story is clear: the incumbent engages in less policy distortion, which makes it easier for voters to screen out racially incompetent types in equilibrium. Polarizing is only beneficial to the incumbent when the next period demands racial competence, since it blurs voter perceptions.

Earlier, I provided evidence that perceptions of racial competency heighten the pressure to act within group constraints. Substantively, this was due to the role of reflected appraisals - the process by which Multiracial identity is defined through others' perceptions - in guiding Multiracial behavior (Khanna 2004, 2010; Sims 2016). Here, polarization provides an avenue through which Multiracial incumbents can blur the lines of racial competency. Therefore, polarization serves not only as an important conduit of perceptions but also diminishes the burden of evidence for Multiracial incumbents. Implication 4 connects reflected appraisals to the work by Davenport (2016) and Davenport, Franco, and Iyengar (2022), showing why uninformed Multiracials are more likely to polarize in order to appear racially competent.

Implication 4. *For Multiracial incumbents, polarization acts as a mechanism of reflected appraisal: by exaggerating policy extremity, uninformed types reduce the burden of proving racial competency.*

Legislative Constraints

A final avenue of increasing perceptions of racial competency is joining social identity-oriented caucuses in the legislature. Canon (1999) shows that identity caucuses serve a primary purpose of constraining legislative behavior, providing resources to coracial constituencies, and ensuring powerful leadership positions for members. Rendleman (2023), focusing on the constraining mechanism of caucuses, highlights that the primary function of institutional organizations is to act as social monitors, pressuring astray members from potentially deviating from coracial goals.[11] Here, I focus on the role identity caucuses have on constraining Multiracial incumbents' ability to distort policy and the influence racial competency has on their willingness to constrain.

Assume that there is some legislative constraint $[-\Psi, \Psi]$ such that $\Psi < \overline{x}^P(\rho)$.[12] An analogous case is supported for $-\Psi > \underline{x}^P(\rho)$. Moreover, let $V_C(\Psi)$ be the voter's continuation value for electing the challenger given the constraint. Up to this point, I have assumed that $d_1 = P$. However, to illustrate a conservative aspect of caucus membership, I assume now that $d_1 = U$. The argument I'm going to make is that caucus membership serves as a signal of racial competency through the channel ρ operates: voters' perceptions. A universal first period provides the most conservative test of this racial competency's role, as voters do not require contextual knowledge ex ante. Multiracial incumbents, therefore, are less likely to engage in caucus participation *a priori*. Yet, Lemma 4 shows that this is not the case.

Lemma 4 (Symmetric Legislative Constraint)**.** *Suppose the legislative constraints are strong* $(\Psi < \overline{x}^P(\rho))$*.*

1. *Assume the incumbent is popular* $(\pi_I > \pi_C)$*.*

11. Lemi (2018) finds the caucus participation for Multiracial politicians is difficult in large part due to this social monitoring mechanism. Because they doubt the racial competence of the Multiracial, caucuses are less likely to allow them to participate.

12. When constraints are weak, the incumbent distorts policy as characterized in Proposition 1.

- *Informed Incumbent: If $\omega \geq 0$ then the incumbent chooses $x_1 = \Psi$. If $\omega < 0$, then the incumbent chooses $x_1 = -\Psi$.*
- *Uninformed Incumbent: The incumbent chooses $x_1 = \Psi$ with probability $1 - F(0)$ and $x = -\Psi$ with probability $F(0)$.*
- *Voter: The voter always re-elects the incumbent.*

2. *Assume the incumbent is unpopular ($\pi_I < \pi_C$).*
   - *Informed Incumbent: If $\omega \geq 0$ then the incumbent chooses $x_1 = \Psi$. If $\omega < 0$, then the incumbent chooses $x_1 = -\Psi$.*
   - *Uninformed Incumbent: The incumbent chooses $x_1 = \Psi$ with probability $\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}\big(1-F(0)\big)$ and $x = -\Psi$ with probability $\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}F(0)$.*
   - *Voter: The voter reelects the incumbent with probability $\mu(\Psi) = \frac{\Psi^2+\sigma^2}{\beta-(1-\rho)\sigma^2-V_C(\Psi)}$.*

Similar to Bils (2023), when the incumbent is popular, caucus constraints diminish the accountability mechanism for voters, since both types of incumbents pool on the constraint and are always re-elected. By contrast, if the incumbent is unpopular, the informed type still selects the constraint in equilibrium. Still, the uninformed type requires an incentive to do so, rather than choosing their optimal payoff, $x_1 = 0$, given that the challenger is ex-ante more likely to be racially competent. This incentive arises when voters mix between re-electing the incumbent and supporting the challenger, making the incumbent indifferent. Driving the voter's re-election probability are perceptions about future racial competency, both indirectly via their continuation value but directly within the probability itself. Perceptions drive Multiracial behavior (Khanna 2004, 2010; Sims 2016), and uncertainty about racial competence increases their willingness to engage in potentially risky political action such as polarization (Davenport 2016; Davenport, Franco, and Iyengar 2022). Therefore, uncertainty raises the salience for uninformed incumbents to constrain themselves *even if the present environment doesn't necessitate it*. In turn, the incumbent mixes between setting $x_1 = \Psi$ with some probability and $x_1 = 0$, which then keeps voters indifferent between retaining the incumbent and electing the challenger.

Voters' perceptions and incumbents' uncertainty about ρ drive Multiracial behavior. Proposition 6 highlights how racial competency mediates the effect of the caucus constraint on the voter's re-election probability

Proposition 6 (Impact of ρ on Voters' Perceptions)**.** *As ρ increases, the re-election probability for the uninformed incumbent increases.*

The results of Proposition 6 indicate that constraints improve the incumbent's re-election probability, and that ρ mediates this effect. By constraining themselves in the present universal period, incumbents ensure against the possibility that an electorate requiring racial competency arises tomorrow. In this sense, the constraint functions as a coracial commitment: a costly signal of racial competence even when the incumbent is not inherently competent. For Multiracial incumbents, joining the caucus - even when the immediate policy domain does not require racial competence - enhances the perception that they are racially competent should the next period demand it. By improving one's perceptions, caucus membership also serves as another important mechanism of reflected appraisal. Implication 5 provides evidence of this fact.

Implication 5. *For Multiracial incumbents, racial caucus membership is a mechanism of reflected appraisal: by joining a racial caucus, uninformed types reduce the burden of proving racial competency.*

Although the comparative static suggests that re-election prospects are higher when tomorrow is universal, this effect arises precisely because the incumbent constrained themselves today rather than pursuing their unconstrained policy optimum. The insurance is thus forward-looking: present discipline secures future electoral viability.[13] This also provides a new channel for why incumbents whose values do not align with the caucus otherwise conform to the constraints imposed. Rendleman (2023) shows that caucus membership serves as a racialized social constraint (White and Laird 2020), sanctioning members who deviate from their legislative priorities. The threat of needing to be racially competent in the future also acts as an indirect sanction, not through legislative peers but through voters' perceptions. Multiracials worry about their caucus perceptions (Lemi 2018), but they are also driven by voters' expectations within the electoral context. Thus, caucus membership provides both an institutional (Canon 1999) and electoral utility from participation.

Discussion and Conclusion

In this paper, I have demonstrated that racial competency conditions the behavior of multiracial officeholders. Uninformed politicians lack racial competency but desire to be perceived as such; To credibly signal to voters that they are racially competent, uninformed politicians exaggerate policy. Significantly, uncertainty about racial competency affects both the extent and willingness to posture in equilibrium. Because incumbents are more willing to posture when they are perceived as racially competent ex ante, elections serve as a poor screening mechanism unless constraints are enforced or voters require contextual knowledge in following periods. Incumbents attempt to circumvent this by polarization in competitive elections, although racial competency mitigates the effectiveness of ideology. Finally, although caucus membership effectively constrains policy distortion, it is only through uncertainty about the future need for racial competency.

My model shows that political alignment does not equate to racial competency. Multiracials are more likely to align with their monoracial minority peers (Davenport, Iyengar,

13. Appendix C.2.2 shows the analogous comparative static for $d_1 = P$. While the result is similar, re-election probabilities have a steeper slope when $d_1 = U$ due to the positive σ^2 term in the voter's re-election function.

and Westwood 2022). Such alignment often translates into more ideologically extreme positions (Davenport 2016; Davenport, Franco, and Iyengar 2022). Leslie and Sears (2022) shows that for Black–White Multiracials in particular, the legacy of hypodescent structures political alignment, offering a precise mechanism for why Multiracials are aligned with their minority background. My model highlights how the desire to be perceived as racially competent underlies much of the literature on alignment. Multiracials polarize when uncertainty about the need for racial competency is high. They also participate in racial caucuses to mitigate the risk of being perceived as unaligned in the future. In this way, my model provides a microfoundation for how Multiracials can be politically aligned yet still lack racial competency.

Moreover, the model shows that racial competency is a significant constraint on Multiracial behavior, structuring their actions even when voters face information barriers. In standard models of democratic theory, voters *authorize* politicians to represent them and hold them *accountable* through elections (Pitkin 1967; Young 1997, 2002). Racial competency facilitates representation for disadvantaged groups by acting as a form of insurance: it ensures that the politician will act on behalf of the marginalized group's interest if the deliberative body is unaware of, or unwilling to pursue, those interests (Dovi 2002; Mansbridge 1999, 2009; Williams 1993). What has remained unclear, however, is whether this logic of racial competency extends to Multiracial politicians and their constituencies. Here, I show that though Multiracial politicians have opportunities to exaggerate behavior, racial competency remains a tool for ensuring minority voters receive credible representation.

I qualify Bils (2023) by showing that racial competency functions as a selection mechanism by raising the level of office rents required to lower voter welfare. Within the political economy literature, voter welfare decreases as prior beliefs about an incumbent's competency increase (Bils 2023; Canes-Wrone, Herron, and Shotts 2001). Incumbents are more willing to posture when they are perceived as competent, which leads to worse outcomes for voters (Bils 2023). The findings presented here indicate that this willingness is influenced by uncertainty regarding the necessity of racial competency. By increasing the office rents required, racial competency improves voters' ability to distinguish between competent and incompetent types in equilibrium.

Beyond the application to racial politics in the U.S., this paper has broader implications by examining how ambiguous identities influence the agency relationship between voters and representatives. Although Multiracial politicians embody multiple perspectives and thus appear as "jacks of all trades", the challenge of sustaining reciprocal relationships with diverse constituencies risks rendering them "masters of none". My model clarifies this tension, showing that the very plurality of perspectives that seems to enhance representation can, under electoral pressures, complicate the ability of politicians to demonstrate competence credibly. In this sense, the Multiracial case illuminates not only the dilemmas of descriptive representation in the United States, but also the general problem of how democratic institutions process identity complexity.

Future work should investigate the extent to which the model can be applied to other dual-identity or ambiguous social groups. For example, representatives who hold multiple religious identities may face similar incentives to be perceived as competent in order to secure re-election. The framework also naturally extends to immigrant groups who maintain a strong ethnic identity yet must present themselves as "authentically" American. Beyond these theoretical extensions, future research should empirically test the model's predictions on Multiracial respondents. In particular, the ideological arguments are closely tied to the literature on racial attitudes and Multiracial political behavior (Davenport 2016; Davenport, Franco, and Iyengar 2022). While Davenport (2016) and Davenport, Franco, and Iyengar (2022) demonstrates that psychological dispositions influence Multiracial respondents to be more liberal than their monoracial peers, my model provides a mechanism for evaluating the strategic underpinnings of this behavior. Relatedly, there are avenues to investigate whether Multiracial or other dual-identity politicians engage in distinctive legislative behavior in Congress, particularly in relation to participation in social identity organizations. Ultimately, the model provides a generative framework for examining the formal extensions and empirical implications of Multiracial behavior.

References

Bils, Peter. 2023. "Overreacting and Posturing: How Accountability and Ideology Shape Executive Policies." *Quarterly Journal of Political Science* 18 (2). ISSN: 1554-0626. https://doi.org/10.1561/100.00020177.

Brown, Nadia. 2014. ""It's more than hair… that's why you should care": the politics of appearance for Black women state legislators." *Politics, Groups, and Identities* 2 (3): 295–312.

Butler, Daniel M, and David E Broockman. 2011. "Do politicians racially discriminate against constituents? A field experiment on state legislators." *American Journal of Political Science* 55 (3): 463–477.

Canes-Wrone, Brandice, Michael C. Herron, and Kenneth W. Shotts. 2001. "Leadership and Pandering: A Theory of Executive Policymaking." *American Journal of Political Science* 45 (3): 532. ISSN: 0092-5853. https://doi.org/10.2307/2669237.

Canes-Wrone, Brandice, and Kenneth W. Shotts. 2007. "When do elections encourage ideological rigidity?" *American Political Science Review* 101 (2): 273–288.

Canon, David T. 1999. *Race, Redistricting, and Representation.* University of Chicago Press.

Cho, In-Koo, and David M Kreps. 1987. "Signaling games and stable equilibria." *The Quarterly Journal of Economics* 102 (2): 179–221.

Davenport, Lauren D. 2016. "Beyond Black and White: Biracial Attitudes in Contemporary U.S. Politics." *American Political Science Review,* https://doi.org/10.1017/s0003055415000556.

Davenport, Lauren D., Annie Franco, and Shanto Iyengar. 2022. "Multiracial identity and political preferences." *The Journal of Politics* 84 (1): 620–624.

Davenport, Lauren D., Shanto Iyengar, and Sean J. Westwood. 2022. "Racial identity, group consciousness, and attitudes: A framework for assessing multiracial self-classification." *American Journal of Political Science* 66 (3): 570–586.

Davis, F James. 1991. *Who is black?: One nation's definition.* Penn State Press.

Dawson, Michael C. 1994. *Behind the mule: Race and class in African-American politics.* Princeton University Press.

Dovi, Suzanne. 2002. "Preferable descriptive representatives: Will just any woman, black, or Latino do?" *American Political Science Review* 96 (4): 729–743.

Fenno, Richard. 1978. *Home style: House members in their districts.* Little, Brown Boston.

Fox, Justin, and Matthew C. Stephenson. 2011. "Judicial Review as a Response to Political Posturing." *American Political Science Review* 105 (2): 397–414. ISSN: 0003-0554. https://doi.org/10.1017/s0003055411000116.

Grose, Christian R. 2011. *Congress in black and white: Race and representation in Washington and at home.* Cambridge University Press.

Hardy-Fanta, Carol, Pei-te Lien, Dianne Marie Pinderhughes, and Christine M Sierra. 2013. "Racial and Ethnic Identity of Elected Officials of Color: A Closer Look at a Complex Matter." In *APSA 2013 Annual Meeting Paper, American Political Science Association 2013 Annual Meeting.*

Haynie, Kerry L. 2001. *African American legislators in the American states.* Columbia University Press.

Hero, Rodney E, and Caroline J Tolbert. 1995. "Latinos and substantive representation in the US House of Representatives: Direct, indirect, or nonexistent?" *American Journal of Political Science,* 640–652.

Hochschild, Jennifer, and Vesla Mae Weaver. 2010. ""There's no one as Irish as Barack O'bama": the policy and politics of American multiracialism." *Perspectives on Politics* 8 (3): 737–759.

Khanna, Nikki. 2004. "The role of reflected appraisals in racial identity: The case of multiracial Asians." *Social Psychology Quarterly* 67 (2): 115–131.

———. 2010. ""If you're half black, you're just black": Reflected appraisals and the persistence of the one-drop rule." *The Sociological Quarterly* 51 (1): 96–121.

Lemi, Danielle Casarez. 2018. "Identity and coalitions in a multiracial era: how state legislators navigate race and ethnicity." *Politics, Groups, and Identities* 6:725–742. ISSN: 2156-5503, 2156-5511. https://dx.doi.org/10.1080/21565503.2017.1288144.

———. 2021. "Do voters prefer just any descriptive representative? The case of multiracial candidates." *Perspectives on Politics* 19 (4): 1061–1081.

Leslie, Gregory, Natalie Masuoka, Sarah E. Gaither, and A. Chyei Vinluan Jessica D. Remedios. 2022. "Voter Evaluations of Biracial-Identified Political Candidates." *Social Sciences* 11 (4): 171. ISSN: 2076-0760. https://doi.org/10.3390/socsci11040171.

Leslie, Gregory, and David O Sears. 2022. "The heaviest drop of blood: Black exceptionalism among multiracials." *Political Psychology* 43 (6): 1123–1145.

Levy, Gilat. 2004. "Anti-herding and strategic consultation." *European Economic Review* 48 (3): 503–525. ISSN: 0014-2921. https://doi.org/10.1016/s0014-2921(03)00019-9.

Mansbridge, Jane. 1999. "Should blacks represent blacks and women represent women? A contingent" yes"." *The Journal of politics* 61 (3): 628–657.

———. 2009. "A "selection model" of political representation." *Journal of Political Philosophy* 17 (4): 369–398.

Maskin, Eric, and Jean Tirole. 2004. "The politician and the judge: Accountability in government." *American Economic Review* 94 (4): 1034–1054.

McConnaughy, Corrine M, Ismail K White, David L Leal, and Jason P Casellas. 2010. "A Latino on the ballot: Explaining coethnic voting among Latinos and the response of White Americans." *The Journal of Politics* 72 (4): 1199–1211.

Minta, Michael D, and Nadia Brown. 2014. "Intersecting interests: Gender, race, and congressional attention to women's issues." *Du Bois Review: Social Science Research on Race* 11 (2): 253–272.

Pew. 2015. "Multiracial in America: Proud, Diverse and Growing in Numbers." Accessed April 30, 2025. https://www.pewresearch.org/social-trends/2015/06/11/multiracial-in-america/.

Pitkin, Hanna. 1967. *The concept of representation.*

Prendergast, Canice, and Lars Stole. 1996. "Impetuous Youngsters and Jaded Old-Timers: Acquiring a Reputation for Learning." *Journal of Political Economy* 104 (6): 1105–1134. ISSN: 0022-3808. https://doi.org/10.1086/262055.

Preuhs, Robert R. 2006. "The conditional effects of minority descriptive representation: Black legislators and policy influence in the American states." *The Journal of Politics* 68 (3): 585–599.

Rendleman, Hunter E. 2023. "Bound Together: Racial Peer Effects and Caucus Control in the US Congress."

Rockquemore, Kerry Ann, and David Brunsma. 2002. "Beyond black." *Biracial identity in America.*

Sims, Jennifer Patrice. 2016. "Reevaluation of the influence of appearance and reflected appraisals for mixed-race identity: The role of consistent inconsistent racial perception." *Sociology of Race and Ethnicity* 2 (4): 569–583.

Starr, Paul, and Christina Pao. 2024. "The Multiracial Complication: The 2020 Census and the Fictitious Multiracial Boom." *Sociological Science* 11:1107–1123.

Swain, Carol Miller. 1995. *Black faces, black interests: The representation of African Americans in Congress.* Harvard University Press.

Tate, Katherine. 2004. *Black faces in the mirror: African Americans and their representatives in the US Congress.* Princeton University Press.

U.S. Census Bureau. 2022. "Data Derived from the Decennial Census." https://www.census.gov/library/stories/2021/08/improved-race-ethnicity-measures-reveal-united-states-population-much-more-multiracial.html.

Velasquez-Manoff, Moisés. 2017. "What Biracial People Know." *The New York Times,* https://www.nytimes.com/2017/03/04/opinion/sunday/what-biracial-people-know.html?_r=0.

Wamble, Julian J. 2025. *We Choose You.* Cambridge University Press: Cambridge University Press, February. ISBN: 9781009483162.

Whitby, Kenny J. 2000. *The color of representation: Congressional behavior and black interests.* University of Michigan Press.

White, Ismail K, and Chryl N Laird. 2020. “Steadfast democrats: How social forces shape Black political behavior.” In *Steadfast Democrats.* Princeton University Press.

Williams, Melissa Suzanne. 1993. *Voice, trust and memory: marginalized groups and the failings of liberal representation.* Harvard University.

Young, Iris Marion. 1997. “Deferring group representation.” *Ethnicity and group rights* 39:349–376.

———. 2002. *Inclusion and democracy.* OUP Oxford.

Appendix

Contents

A Proofs of Main Findings

A.1 Second-period Behavior

Lemma 1 (Impact of ρ on Continuation Values). *The voter's continuation value from electing the challenger is strictly increasing in ρ, and converges to 0 as ρ approaches 1.*

Proof of Lemma 1. let

$$V(C) = -[1-\pi_C](1-\rho)\sigma^2.$$

To compute the partial derivative of $V(C)$ with respect to ρ:

$$\frac{\partial V(C)}{\partial \rho} = -[1-\pi_C]\sigma^2 \cdot \frac{\partial}{\partial \rho}(1-\rho).$$

Since

$$\frac{\partial}{\partial \rho}(1-\rho) = -1,$$

It follows that

$$\frac{\partial V(C)}{\partial \rho} = -[1-\pi_C]\sigma^2 \cdot (-1) = [1-\pi_C]\sigma^2.$$

Because $\pi_C \in (0,1)$ implies that $1-\pi_C > 0$ and since $\sigma^2 > 0$, we have

$$\frac{\partial V(C)}{\partial \rho} > 0.$$

Thus, $V(C)$ is strictly increasing in ρ.

Next, consider the limit as ρ approaches 1:

$$\lim_{\rho \to 1} V(C) = \lim_{\rho \to 1} \Big(-[1-\pi_C](1-\rho)\sigma^2\Big).$$

Since

$$\lim_{\rho \to 1}(1-\rho) = 0,$$

we obtain

$$\lim_{\rho \to 1} V(C) = -[1-\pi_C]\sigma^2 \cdot 0 = 0.$$

Hence, as ρ approaches 1, the continuation value $V(C)$ converges to 0. □

Lemma 2 (Voter Decision Rule). *After observing first-period policy x_1 in equilibrium, the voter (1) reelects the incumbent if $\tilde{\pi}(x_1) > \pi_C$, (2) reelects the challenger if $\tilde{\pi}(x_1) < \pi_C$, and (3) is indifferent if $\tilde{\pi}(x_1) = \pi_C$.*

Proof of Lemma 2. Define the voter's payoff difference as

$$\Delta(x_1) \;=\; V(I \mid x_1) - V(C)$$

We want to show that $\Delta(x_1) > 0$ if and only if $\tilde{\pi}(x_1) > \pi_C$.

Let

$$V(I \mid x_1) \;=\; -\big[\,1 - \tilde{\pi}(x_1)\big](1-\rho)\,\sigma^2, \qquad V(C) \;=\; -\big[\,1 - \pi_C\big](1-\rho)\,\sigma^2.$$

Then,

$$\Delta(x_1) = -\big[\,1 - \tilde{\pi}(x_1)\big](1-\rho)\,\sigma^2 - \big\{-\big[1 - \pi_C\big](1-\rho)\sigma^2\big\}.$$

Combining like terms results in,

$$\Delta(x_1) = (1-\rho)\sigma^2\big\{[1-\pi_C] - [1-\tilde{\pi}(x_1)]\big\}.$$

Simplifying further,

$$\Delta(x_1) = (1-\rho)\sigma^2[\tilde{\pi}(x_1) - \pi_C].$$

Since $(1-\rho)\sigma^2 > 0$, the sign of $\Delta(x_1)$ is entirely determined by $[\tilde{\pi}(x_1) - \pi_C]$. Therefore, $\Delta(x_1) > 0 \iff \tilde{\pi}(x_1) > \pi_C$.

An analogous case can be shown for $\Delta(x_1) < 0 \iff \tilde{\pi}(x_1) < \pi_C$ and $\Delta(x_1) = 0 \iff \tilde{\pi}(x_1) = \pi_C$. □

A.2 Complete Information Benchmark

Remark 1. *Under the complete information benchmark, there is a subgame perfect Nash equilibrium in which the high-type incumbent always selects $x_1 = \omega_1$, and the low-type incumbent selects*

$$x_1 = \begin{cases} \omega_1, & \text{if } d_1 = U, \\ 0, & \text{if } d_1 = P. \end{cases}$$

The voter always reelects the high-type and rejects the low type.

Proof of Remark 1. It is straightforward to show that both types of incumbents never deviate when $d_1 = U$. Thus, I focus on the uninformed incumbent when $d_1 = P$. In this case, suppose that the incentive compatibility constraint is given by

$$\underbrace{\beta - \sigma^2}_{\text{Equilibrium Payoff}} \quad - \underbrace{[1-\pi_C]\sigma^2}_{V(C)} \quad \geq \quad \underbrace{2\beta - (2-\rho)\sigma^2}_{\text{Deviation Payoff}}.$$

We begin by writing the inequality as

$$\left(\beta - \sigma^2\right) - [1-\pi_C]\,\sigma^2 \geq 2\beta - (2-\rho)\sigma^2.$$

Subtracting $\beta - \sigma^2$ from both sides gives

$$-[1-\pi_C]\sigma^2 \geq \Big[2\beta - (2-\rho)\sigma^2\Big] - \left(\beta - \sigma^2\right).$$

Simplify the right-hand side:

$$\begin{aligned} 2\beta - (2-\rho)\sigma^2 - (\beta - \sigma^2) &= 2\beta - (2-\rho)\sigma^2 - \beta + \sigma^2 \\ &= \beta - \Big[(2-\rho)\sigma^2 - \sigma^2\Big] \\ &= \beta - \Big[(2-\rho-1)\sigma^2\Big] \\ &= \beta - (1-\rho)\sigma^2. \end{aligned}$$

Thus, the inequality becomes

$$-[1-\pi_C]\sigma^2 \geq \beta - (1-\rho)\sigma^2.$$

Multiplying both sides by -1 (which reverses the inequality) yields

$$[1-\pi_C]\sigma^2 \leq (1-\rho)\sigma^2 - \beta.$$

Adding β to both sides,

$$[1-\pi_C]\sigma^2 + \beta \leq (1-\rho)\sigma^2,$$

and then subtracting $[1-\pi_C]\sigma^2$ from both sides gives

$$\beta \leq (1-\rho)\sigma^2 - [1-\pi_C]\sigma^2.$$

Factoring σ^2 on the right-hand side,

$$\beta \leq \big[(1-\rho) - (1-\pi_C)\big]\sigma^2.$$

Since

$$(1-\rho) - (1-\pi_C) = \pi_C - \rho,$$

we obtain

$$\beta \leq (\pi_C - \rho)\sigma^2,$$

or equivalently,

$$\pi_C\sigma^2 - \rho\sigma^2 \geq \beta.$$

Under the assumption that $\pi_C \in (0,1)$ and $\sigma^2 > 0$, the right-hand side $\pi_C\sigma^2 - \rho\sigma^2$ is strictly positive for any $\rho < \pi_C$. Hence, there exists a range of β satisfying

$$\beta \leq (\pi_C - \rho)\sigma^2.$$

Therefore, when $d_1 = P$, the uninformed incumbent has no incentive to deviate from their equilibrium strategy. □

A.3 Uncertainty about Racial Competency

A.3.1 Low-value Office

Remark 2. *If office rents are low so that* $\beta \leq \pi_C\sigma^2 - \rho\sigma^2$*, then there exists a perfect Bayesian equilibrium in which the high-type incumbent selects* $x_1 = \omega_1$ *and the low-type incumbent selects* $x_1 = 0$*.*

Proof of Remark 2. Assume that there exists a perfect Bayesian equilibrium when office rents are low. In such an equilibrium, the informed incumbent chooses

$$x_1 = \omega_1,$$

while the uninformed incumbent chooses

$$x_1 = 0.$$

By weak consistency, when a voter observes $x_1 = \omega_1$, she assigns probability 1 to the incumbent being high-type. Moreover, by sequential rationality, the voter's updated belief, $\tilde{\pi}_I(x_1)$, satisfies

$$\tilde{\pi}_I(x_1) > \pi_C,$$

ensuring that re-election occurs only when the incumbent is perceived as an informed type.

To complete the argument, we must show that the uninformed type has no profitable deviation to mimic the informed type. Under this condition, the incentive compatibility constraint simplifies to:

$$\pi_C\sigma^2 \geq \beta.$$

This inequality holds for β sufficiently small, ensuring that the low-type's deviation is not profitable. Thus, a separating, first-best equilibrium exists when the policy domain is particular. □

A.3.2 High-value Office

Proposition 1 (Perfect Bayesian Equilibria with High Office Rents). *The set of Perfect Bayesian Equilibria of the model is characterized as follows:*

1. ***Voter strategy.***
   - *If* $x_1 \in (\underline{x}^P(\rho),\ \overline{x}^P(\rho))$*, the voter rejects the incumbent.*
   - *If* $x_1 \leq \underline{x}^P(\rho)$ *or* $x_1 \geq \overline{x}^P(\rho)$*, the voter re-elects the incumbent.*
2. ***Informed incumbent.***
   - *If* $\omega_1 \in [0, \omega^P(\rho))$*, the informed type overreacts by setting* $x_1 = \overline{x}^P(\rho)$*.*
   - *If* $\omega_1 \in (-\omega^P(\rho), 0]$*, the informed type overreacts by setting* $x_1 = \underline{x}^P$*.*
   - *If* $|\omega_1| > \omega^P(\rho)$*, the informed type plays the first-best by choosing* $x_1 = \omega_1$*.*
3. ***Uninformed incumbent.***
   - *With probability* $\overline{\sigma} \in [0, \min\{1, \overline{\Sigma}\}]$*, the uninformed type postures right by choosing* $x_1 = \overline{x}^P(\rho)$*.*
   - *With probability* $\underline{\sigma} \in [0, \min\{1-\overline{\sigma}, \underline{\Sigma}\}]$*, the uninformed type postures left by choosing* $x_1 = \underline{x}^P(\rho)$*.*
   - *With residual probability* $1 - \underline{\sigma} - \overline{\sigma}$*, the uninformed type plays the first-best by setting* $x_1 = 0$*.*

Off the path of play, assume the voter updates their belief with probability 1 that the incumbent is uninformed.

Proof of Proposition 1.

Uninformed Incumbent. The uninformed incumbent's utility is

$$-x^2 - \sigma^2.$$

Choosing any $x_1 \in (\underline{x}^P(\rho), \overline{x}^P(\rho)) \setminus \{0\}$ yields a payoff that is weakly worse than choosing $x_1 = 0$. Moreover, selecting any $x_1 > \overline{x}^P(\rho)$ or $x_1 < \underline{x}^P(\rho)$ (and getting re-elected) is strictly worse than selecting $x_1 = \overline{x}^P(\rho)$ or $x_1 = \underline{x}^P(\rho)$. Thus, both $\underline{x}^P(\rho)$ and $\overline{x}^P(\rho)$ render the uninformed type indifferent among the three actions:

$$x_1 = \overline{x}^P(\rho), \quad x_1 = \underline{x}^P(\rho), \quad \text{and} \quad x_1 = 0.$$

Consequently, the uninformed incumbent mixes over these three actions.

Informed Incumbent. For the informed incumbent, if $\omega_1 \geq \overline{x}^P(\rho)$ or $\omega_1 \leq \underline{x}^P(\rho)$, the optimal policy is to choose $x_1 = \omega_1$. Now, partition the state space such that for $\omega_1 \in [0, \overline{x}^P(\rho))$ the optimal policy to secure re-election is $x_1 = \overline{x}^P(\rho)$. If the informed type were to choose their bliss point $x_1 = \omega_1$, they would be removed from office. Thus, their payoffs are:

$$\text{If removed: } \beta - (1-\pi_C)\sigma^2, \qquad \text{if re-elected: } -(\overline{x}^P(\rho) - \omega_1)^2 + 2\beta.$$

The distortion cost $(\overline{x}^P(\rho) - \omega_1)^2$ is decreasing in ω_1 since

$$\frac{\partial\left[(\overline{x}^P(\rho) - \omega_1)^2 + 2\beta\right]}{\partial \omega_1} = -2(\overline{x}^P(\rho) - \omega_1) < 0.$$

Hence, if the type with $\omega_1 = 0$ prefers $x_1 = \overline{x}^P(\rho)$, then every $\omega_1 \in (0, \overline{x}^P(\rho))$ also prefers $x_1 = \overline{x}^P(\rho)$. To verify this, consider:

$$\begin{aligned}
-x^2 + 2\beta &> \beta - (1-\pi_C)\sigma^2, \\
-x^2 + \beta &> -(1-\pi_C)\sigma^2, \\
(1-\pi_C)\sigma^2 + \beta &> x^2, \\
(1-\pi_C)\sigma^2 + \beta &> \beta - \pi_C\sigma^2 + \rho\sigma^2, \\
\sigma^2 - \pi_C\sigma^2 &> \pi_C\sigma^2 + \rho\sigma^2, \\
\sigma^2 &> \rho\sigma^2
\end{aligned}$$

which holds for all $\rho \in [0,1)$. A similar argument applies for $\omega_1 \in (\underline{x}^P(\rho), 0)$ when choosing $x_1 = \underline{x}^P(\rho)$. Thus, the $\omega_1 = 0$ type prefers to select $x_1 = \overline{x}^P(\rho)$ (and be re-elected) over the bliss point $x_1 = 0$.

Voter Decision. A voter re-elects the incumbent if $x_1 \geq \overline{x}^P(\rho)$ or $x_1 \leq \underline{x}^P(\rho)$, and elects the challenger if $x_1 \in (\underline{x}^P(\rho), \overline{x}^P(\rho))$. By D1, any policy $x_1 \in (\underline{x}^P(\rho), \overline{x}^P(\rho)) \setminus \{0\}$ is off the equilibrium path, so that the voter's posterior satisfies $\tilde{x}_I(x_1) \leq \pi_C$ and the challenger is elected. We set $\tilde{x}_I(x_1) = 0$ for such x_1. Since the uninformed type selects $x_1 = 0$, the voter's posterior is $\tilde{x}_I(0) = 0 < \pi_C$. In contrast, only the informed type selects $x_1 > \overline{x}^P(\rho)$ or $x_1 < \underline{x}^P(\rho)$; in those cases, $\tilde{x}_I(x_1) = 1 > \pi_C$. Finally, when $x_1 = \overline{x}^P(\rho)$, it is required that

$$\tilde{\pi}_I(x_1) \geq \pi_C,$$

so that the voter re-elects the incumbent. By Lemma 2, we have

$$\tilde{\pi}_I(x_1) = \frac{(F(\omega^P(\rho)) - F(0))\,\pi_I}{\pi_I\big(F(\omega^P(\rho)) - F(0)\big) + (1-\pi_C)\,\overline{\Sigma}} \geq \pi_C$$

which can be rearranged to

$$\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}\Big(F(\omega^P(\rho)) - F(0)\Big) \geq \overline{\Sigma}.$$

A similar argument holds for $x_1 = \underline{x}^P(\rho)$. □

Remark 3. *The equilibria identified in Proposition 1 survive D1.*

Proof of Remark 3. .

Step 1: Sending off-the-path Message $\tilde{x}^P(\rho)$.

An arbitrary incumbent's typ is given by $\tau \in \mathbb{R} \cup \{\phi\}$. Define $D_\sigma(\tau, x)$ as the set of mixed best responses (MBR) where the incumbent is strictly better off deviating to an new policy x and getting re-elected with probability μ versus following equilibrium x in a PBE σ. Let $D^o_\sigma(\tau, x)$ as the set of strategies that make τ indifferent. If $\tilde{x}^P(\rho)$ is off-path, D1 requires putting probability 0 on τ-type if there exist some τ'-type such that

$$D_\sigma(\tau, \tilde{x}^P(\rho)) \cup D^o_\sigma(\tau, \tilde{x}^P(\rho)) \subseteq D_\sigma(\tau', \tilde{x}^P(\rho)).$$

Setting $\omega_1 \geq \overline{x}^P(\rho)$ and $\omega_1 \leq \underline{x}^P(\rho)$, the informed incumbent never faces an incentive to deviate since they receive their highest payoff.

Examining the $\omega_1 \in [0, \overline{x}^P(\rho)]$ case for the Informed incumbent. The informed type selects $x_1 = \overline{x}^P(\rho)$ and their payoff

$$-(\overline{x}^P(\rho) - \omega_1) + 2\beta.$$

If they deviate to some $x_1 = \tilde{x}^P(\rho)$, their payoff given $\mu(\overline{x}^P(\rho))$ is

$$-(\tilde{x}^P(\rho) - \omega_1) + \beta + \mu(\tilde{x}^P(\rho))\beta - (1 - \mu(\tilde{x}^P(\rho)))[(1 - \pi_C)\sigma^2].$$

Therefore, the incentive compatibility constraint is

$$\begin{aligned}
&- \big(\tilde{x}^P(\rho) - \omega_1\big) + \beta + \mu\big(\tilde{x}^P(\rho)\big)\beta - \big(1 - \mu\big(\tilde{x}^P(\rho)\big)\big)\big[(1 - \pi_C)\sigma^2\big] \\
&\qquad > -\big(\overline{x}^P(\rho) - \omega_1\big) + 2\beta \\
&\iff \mu\big(\tilde{x}^P(\rho)\big) > \frac{(\tilde{x}^P(\rho) - \omega_1) - (\overline{x}^P(\rho) - \omega_1) + \beta + (1 - \pi_C)\sigma^2}{\beta + (1 - \pi_C)\sigma^2}.
\end{aligned}$$

Solving the first order condition results in

$$
\begin{aligned}
\frac{\partial\big(\tilde{x}^P(\rho)-\omega_1\big)^2}{\partial\omega_1} &= 2\big(\tilde{x}^P(\rho)-\omega_1\big)(-1)\\
\frac{\partial\big(\overline{x}^P(\rho)-\omega_1\big)^2}{\partial\omega_1} &= 2\big(\overline{x}^P(\rho)-\omega_1\big)(-1)\\
\iff \quad \frac{-2\big(\tilde{x}^P(\rho)-\omega_1\big)+2\big(\overline{x}^P(\rho)-\omega_1\big)}{\beta+(1-\pi_C)\sigma^2}&\\
\iff \quad \frac{2\big(\overline{x}^P(\rho)-\tilde{x}^P(\rho)\big)}{\beta+(1-\pi_C)\sigma^2} &> 0.
\end{aligned}
$$

So, ω_1 is minimized at $\omega_1 = 0$, and D1 requires placing a probability of 0 on any deviation from the $\omega_1 \in (0, \overline{x}^P(\rho))$ type as

$$
D_\sigma(\omega_1 \in (0, \overline{x}^P(\rho)), \tilde{x}^P(\rho)) \subseteq D_\sigma(\omega_1 = 0, \tilde{x}^P(\rho)).
$$

Next consider the incentive for the $\omega_1 \in (\underline{x}^P(\rho), 0)$ type to deviate to some $\tilde{x}^P(\rho) \in (0, \overline{x}^P(\rho))$. Then,

$$
\mu(\tilde{x}^P(\rho)) > \frac{(\tilde{x}^P(\rho)-\omega_1)-(\underline{x}^P(\rho)-\omega_1)+\beta+(1-\pi_C)\sigma^2}{\beta+(1-\pi_C)\sigma^2}.
$$

Taking the partial derivative WRT ω_1,

$$
\begin{aligned}
\frac{\partial(\text{RHS})}{\partial\omega_1} &= \frac{-2\big(\tilde{x}^P(\rho)-\omega_1\big)+2\big(\underline{x}^P(\rho)-\omega_1\big)}{\beta+(1-\pi_C)\sigma^2}\\
&= \frac{2\big(\underline{x}^P(\rho)-\tilde{x}^P(\rho)\big)}{\beta+(1-\pi_C)\sigma^2} < 0.
\end{aligned}
$$

as $\underline{x}^P(\rho) < 0$. Therefore, as ω_1 increases in $(\underline{x}^P(\rho), 0)$, the incentive to follow equilibrium behavior decreases. Because $\omega_1 \to 0$ converges to $\omega_1 = 0$ payoff, D1 requires placing a probability 0 on any $\tilde{x}^P(\rho) \in (0, \overline{x}^P(\rho))$ deviation coming from a $\omega_1 \in (\underline{x}^P(\rho), 0)$ type. An analogous argument shows the case for $\tilde{x}^P(\rho) \in (\underline{x}^P(\rho), 0)$. Therefore, D1 requires placing a probability 0 on any $\omega_1 \in (\underline{x}^P(\rho),\ \overline{x}^P(\rho)) \setminus \{0\}$ type.

Examining the $\omega_1 \in [0, \overline{x}^P(\rho)]$ case for the Uninformed incumbent. Now for the uninformed type's incentive to choose some $\tilde{x}^P(\rho) \in (0, \overline{x}^P(\rho))$. Their equilibrium payoff is

$$
-\overline{x}^P(\rho)^2 - (2-\rho)\sigma^2 + 2\beta.
$$

If they deviate to $x_1 = \tilde{x}^P(\rho)$,

$$-\tilde{x}^P(\rho)^2 - \sigma^2 + \beta + \mu(\tilde{x}^P(\rho))(\beta - \sigma^2) - (1 - \mu(\tilde{x}^P(\rho))[(1 - \pi_C)\sigma^2].$$

The incentive compatibility constraint is

$$\begin{aligned}
&- \tilde{x}^P(\rho)^2 - \sigma^2 + \beta + \mu\big(\tilde{x}^P(\rho)\big)(\beta - \sigma^2) - \big(1 - \mu\big(\tilde{x}^P(\rho)\big)\big)\big[(1 - \pi_C)\sigma^2\big] \\
&\qquad > -\overline{x}^P(\rho)^2 - (2 - \rho)\sigma^2 + 2\beta \\
&= \mu\big(\tilde{x}^P(\rho)\big)(\beta - \sigma^2) - \big(1 - \mu\big(\tilde{x}^P(\rho)\big)\big)\big[(1 - \pi_C)\sigma^2\big] \\
&\qquad > -\tilde{x}^P(\rho)^2 - \overline{x}^P(\rho)^2 + \sigma^2 - (2 - \rho)\sigma^2 + \beta \\
&= \frac{\mu\big(\tilde{x}^P(\rho)\big)\big(\beta - \sigma^2 + (1 - \pi_C)\sigma^2\big)}{\beta - \sigma^2 + (1 - \pi_C)\sigma^2} \\
&\qquad > \frac{\tilde{x}^P(\rho)^2 - \overline{x}^P(\rho)^2 - (2 - \rho)\sigma^2 + \beta + (1 - \pi_C)\sigma^2}{\beta - \sigma^2 + (1 - \pi_C)\sigma^2} \\
&\iff \mu\big(\tilde{x}^P(\rho)\big) > \frac{\tilde{x}^P(\rho)^2 - \overline{x}^P(\rho)^2 - (2 - \rho)\sigma^2 + \beta + (1 - \pi_C)\sigma^2}{\beta - \sigma^2 + (1 - \pi_C)\sigma^2} \\
&= \mu\big(\tilde{x}^P(\rho)\big) > \frac{\tilde{x}^P(\rho)^2 - \overline{x}^P(\rho)^2 - \sigma^2 + \rho\sigma^2 + \beta + (1 - \pi_C)\sigma^2}{\beta - \sigma^2 + (1 - \pi_C)\sigma^2} \\
&= \mu\big(\tilde{x}^P(\rho)\big) > \frac{\tilde{x}^P(\rho)^2 - \overline{x}^P(\rho)^2 + \rho\sigma^2 + \beta - \pi_C\sigma^2}{\beta - \pi_C\sigma^2}.
\end{aligned}$$

To show that the uninformed type is willing to deviate for lower $\mu(\tilde{x}^P(\rho))$ than the $\omega_1 = 0$ type, set the incentive compatibility to

$$\frac{\big(\tilde{x}^P(\rho) - \omega_1\big) - \big(\overline{x}^P(\rho) - \omega_1\big) + \beta + (1 - \pi_C)\sigma^2}{\beta + (1 - \pi_C)\sigma^2} > \frac{\tilde{x}^P(\rho)^2 - \overline{x}^P(\rho)^2 + \rho\sigma^2 + \beta - \pi_C\sigma^2}{\beta - \pi_C\sigma^2}.$$

Setting $\omega_1 = 0$ reduces incentive constraint reduces to

$$\frac{\tilde{x}^P(\rho) - \overline{x}^P(\rho) + \beta + (1 - \pi_C)\sigma^2}{\beta + (1 - \pi_C)\sigma^2} > \frac{\tilde{x}^P(\rho)^2 - \overline{x}^P(\rho)^2 + \rho\sigma^2 + \beta - \pi_C\sigma^2}{\beta - \pi_C\sigma^2}.$$

Let $y = \tilde{x}^P(\rho) - \overline{x}^P(\rho)$. Then,

$$(\beta - \pi_C\sigma^2)(y + \beta + (1 - \pi_C)\sigma^2) > (\beta + (1 - \pi_C)\sigma^2)(y + \beta + \rho\sigma^2 - \pi_C\sigma^2).$$

This simplifies to

$$(\beta - \pi_C\sigma^2)(\beta + (1-\pi_C)\sigma^2) > \sigma^2 y + \rho\sigma^2 + (\beta + (1-\pi_C)\sigma^2)(\beta - \pi_C\sigma^2).$$

Further simplification results in

$$-\sigma^2 y - \rho\sigma^2 > (\beta + (1-\pi_C)\sigma^2)(2\beta - \pi_C\sigma^2) - (\beta - \pi_C\sigma^2)(\beta + (1-\pi_C)\sigma^2).$$

Therefore,

$$\sigma^2(\overline{x}^P(\rho) - \tilde{x}^P(\rho)) - \rho\sigma^2 > 0.$$

Because $\sigma^2 > 0$ and $\overline{x}^P(\rho) - \tilde{x}^P(\rho) > 0$, the incentive constraint holds for ρ sufficiently small. An analogous argument is clear for $\tilde{x}^P(\rho) \in (\underline{x}^P(\rho), 0)$. Therefore, if the voter puts probability 0 on an off-path policy choice $\tilde{x}^P(\rho) \in (\underline{x}^P(\rho),\ \overline{x}^P(\rho))$ coming from an uninformed type, these equilibria survive D1.

Step 2. Strategies and Beliefs form a Perfect Bayesian Equilibrium.

To show these strategies and beliefs form a Perfect Bayesian Equilibrium, I begin with the uninformed type.

Assume $d_1 = P$. Then, their utility is

$$-x^2 - \sigma^2$$

and choosing some $x_1 \in (\underline{x}^P(\rho),\ \overline{x}^P(\rho)) \setminus \{0\}$ results in a strictly worse payoff than selection $x_1 = 0$. Moreover, choosing some $x_1 > \overline{x}^P(\rho)$ or $x_1 < \underline{x}^P(\rho)$ and getting re-elected is strictly worse than setting $x_1 = \overline{x}^P(\rho)$ or $x_1 = \underline{x}^P(\rho)$. Both policy bounds make the uninformed type indifferent between $x_1 = \overline{x}^P(\rho)$, $x_1 = \underline{x}^P(\rho)$, and $x_1 = 0$. As such, the uninformed type mixes over the three actions.

For the informed type. If $\omega_1 \geq \overline{x}^P(\rho)$ or $\omega_1 \leq \underline{x}^P(\rho)$, the optimal policy choice is to select $x_1 = \omega_1$. Now, restrict $\omega_1 \in [0, \overline{x}^P(\rho))$. The optimal policy for re-election is $x_1 = \overline{x}^P(\rho)$. If the informed incumbent selects their bliss point, then $x_1 = \omega_1$ and they are removed from office. Their payoffs are

$$\beta - (1-\pi_C)\sigma^2$$

for removal, and

$$-(\overline{x}^P(\rho)-\omega_1)^2+2\beta$$

for re-election. The cost of selecting $\overline{x}^P(\rho)$ is decreasing in ω_1 such that

$$\frac{\partial(\overline{x}^P(\rho)-\omega_1)^2+2\beta}{\partial\omega_1}=-2(\overline{x}^P(\rho)-\omega_1)<0.$$

Therefore, if $\omega_1=0$ type prefers $\overline{x}^P(\rho)$ then so must all $\omega_1\in(0,\overline{x}^P(\rho))$ types. Then,

$$-\overline{x}^P(\rho)^2+2\beta>\beta-(1-\pi_C)\sigma^2 \iff \sigma^2>\rho\sigma^2.$$

where $\overline{x}^P(\rho)^2=\beta-\pi_C\sigma^2+\rho\sigma^2$. This holds for $\rho\in[0,1)$. The $\omega_1\in(\underline{x}^P(\rho),0)$ case is analogous.

As a result, the $\omega_1=0$ type prefers to select $x_1=\overline{x}^P(\rho)$ and get re-elected over $x_1=0$. because the $\omega_1=0$ type is willing, comparative statics show that all other $\omega_1\in(0,\overline{x}^P(\rho))$ types are also willing The loss from policy distortion is not strong enough to encourage selecting $x_1=\omega_1$ for $\omega_1\in[0,\overline{x}^P(\rho))$.

Finally, a voter must be willing to re-elect an incumbent following $x_1\geq\overline{x}^P(\rho)$ or $x_1\leq\underline{x}^P(\rho)$ and elect the challenger following $x_1\in(\underline{x}^P(\rho),\ \overline{x}^P(\rho))$.

From D1, a policy $x_1\in(\underline{x}^P(\rho),\ \overline{x}^P(\rho)\setminus\{0\})$ is off-the-path. Therefore, $\tilde{\pi}_I(x_1)\leq\pi_C$, and the voter elects the challenger. By D1, let $\tilde{\pi}_I(x_1)=0$ for $x_1\in(\underline{x}^P(\rho),\ \overline{x}^P(\rho)\setminus\{0\})$. The uninformed type selects $x_1=0$, so voters' posterior becomes $\tilde{\pi}_I(x_1)=0<\pi_C$. $x_1>\overline{x}^P(\rho)$ and $x_1<\underline{x}^P(\rho)$ are only selected by the informed incumbent. Under this assumption $\tilde{\pi}_I(x_1)=1>\pi_C$.

Let $x_1=\overline{x}^P(\rho)$. In this case, $\tilde{\pi}_I(x_1)\geq\pi_C$. From Bayes Rule,

$$\tilde{\pi}_I(x_1)=\frac{\Pr(x_1\mid\theta_I=H,d_1=P)\cdot\pi_I}{\Pr(x_1\mid\theta_I=H,d_1=P)\cdot\pi_I+(1-\pi_I)\Pr(x_1\mid\theta_I=L,d_1=P)}.$$

Let $\Pr(x_1\mid\theta_I=H,d_1=P)=F(\omega^P(\rho))-F(0)$ and $\Pr(x_1\mid\theta_I=L,d_1=P)=\overline{\Sigma}$. Then,

$$\tilde{\pi}_I(x_1)=\frac{\big[F(\omega^P(\rho))-F(0)\big]\cdot\pi_I}{\big[F(\omega^P(\rho))-F(0)\big]\cdot\pi_I+(1-\pi_I)\overline{\Sigma}}\ \geq\ \pi_C$$

$$\iff \frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}\big(F(\omega^P(\rho))-F(0)\big)\ \geq\ \overline{\Sigma}.$$

A similar argument is made for $x_1=\underline{x}^P(\rho)$. □

A.4 Voter Welfare

Remark 4. *Voter welfare satisfies:*

1. *Increasing the prior belief that the challenger is informed always increases welfare:*

$$\frac{\partial W}{\partial \pi_C} > 0.$$

2. *Increasing the prior belief that the incumbent is informed decreases welfare whenever*

$$\beta > (2-\rho)\sigma^2.$$

Proof of Remark 4. Let voter welfare be defined as

$$\begin{aligned} W := \pi_I \Big\{ \rho \Big[\int_{-\omega^P(\rho)}^{0} -\big(\underline{x}^P(\rho)-\omega\big)^2 f(\omega)\, d\omega + \int_0^{\omega^P(\rho)} -\big(\overline{x}^P(\rho)-\omega\big)^2 f(\omega)\, d\omega \Big] \\ + (1-\rho)\Big[\int_{-\omega^P(\rho)}^{0} -\big(\underline{x}^P(\rho)-\omega\big)^2 f(\omega)\, d\omega + \int_0^{\omega^P(\rho)} -\big(\overline{x}^P(\rho)-\omega\big)^2 f(\omega)\, d\omega \Big]\Big\} \\ + (1-\pi_I)\Big\{ \rho \Big[\int_{-\omega^P(\rho)}^{0} -\big(\underline{x}^P(\rho)-\omega\big)^2 f(\omega)\, d\omega + \int_0^{\omega^P(\rho)} -\big(\overline{x}^P(\rho)-\omega\big)^2 f(\omega)\, d\omega \Big] + (1-\rho)\big[-\sigma^2-(1-\pi_C)\sigma^2\big]\Big\}. \end{aligned}$$

1. $\frac{\partial W}{\partial \pi_C} > 0$.

The derivative with respect to π_C is

$$(1-\pi_C)(1-\rho)\sigma^2 > 0.$$

As π_C increases, both $\omega^P(\rho)$ and $\overline{x}^P(\rho)$ decrease. This decreases the extent to which the incumbent will overreact. Moreover, it also decreases $\overline{\Sigma}$ and $\underline{\Sigma}$, which decreases the incumbents' willingness to posture. Therefore, increasing π_C increases voter welfare.

2. $\frac{\partial W}{\partial \pi_I} < 0$.

The derivative with respect to π_I is

$$(1-\rho)\big[\int_{-\omega^P(\rho)}^{0} -\big(\underline{x}^P(\rho)-\omega\big)^2 f(\omega)\, d\omega + \int_0^{\omega^P(\rho)} -\big(\overline{x}^P(\rho)-\omega\big)^2 f(\omega)\, d\omega + (2-\pi_C)\sigma^2\big].$$

This term is negative with respect to π_C when $\beta > 2\sigma^2 - \rho\sigma^2$.

□

B Comparative Statics

B.1 Impact of Racial Competency on Policy Extremity

Proposition 2 (Impact of ρ on Policy Extremity)**.** *An increase in the probability that the second-period policy domain is universal leads to an increase in the extremity of the policy choice; that is,*

$$\frac{\partial \overline{x}^P(\rho)}{\partial \rho} > 0 \quad \textit{and} \quad \frac{\partial \underline{x}^P(\rho)}{\partial \rho} < 0.$$

Furthermore, as $\rho \to 1$,

$$\lim_{\rho \to 1} \overline{x}^P(\rho) = \sqrt{\beta - (\pi_C - 1)\sigma^2} \quad \textit{and} \quad \lim_{\rho \to 1} \underline{x}^P(\rho) = -\sqrt{\beta - (\pi_C - 1)\sigma^2}.$$

Proof of Proposition 2. Assume the equilibrium policy positions are given by

$$\overline{x}^P(\rho) = \sqrt{\beta - \pi_C\sigma^2 + \rho\,\sigma^2} \quad \text{and} \quad \underline{x}^P(\rho) = -\sqrt{\beta - \pi_C\sigma^2 + \rho\,\sigma^2}.$$

Define

$$g(\rho) = \beta - \pi_C\sigma^2 + \rho\,\sigma^2.$$

Then we can write

$$\overline{x}^P(\rho) = \big(g(\rho)\big)^{1/2}.$$

Applying the chain rule,

$$\frac{d\overline{x}^P(\rho)}{d\rho} = \frac{d}{d\rho}\Big(g(\rho)^{1/2}\Big) = \frac{1}{2}g(\rho)^{-1/2} \cdot \frac{dg(\rho)}{d\rho}.$$

Since

$$\frac{dg(\rho)}{d\rho} = \sigma^2,$$

we have

$$\frac{\partial \overline{x}^P(\rho)}{\partial \rho} = \frac{1}{2}\big(\beta - \pi_C\sigma^2 + \rho\,\sigma^2\big)^{-1/2}\sigma^2 > 0.$$

Similarly, for the lower policy position,

$$\underline{x}^P(\rho) = -\sqrt{\beta - \pi_C\sigma^2 + \rho\,\sigma^2},$$

so that

$$\frac{\partial \underline{x}^P(\rho)}{\partial \rho} = -\frac{1}{2}\big(\beta - \pi_C\sigma^2 + \rho\,\sigma^2\big)^{-1/2}\sigma^2 < 0.$$

Taking the limit as $\rho \to 1$, we obtain

$$\lim_{\rho \to 1} \overline{x}^P(\rho) = \sqrt{\beta - \pi_C\sigma^2 + \sigma^2} = \sqrt{\beta - (\pi_C - 1)\sigma^2},$$

and

$$\lim_{\rho \to 1} \underline{x}^P(\rho) = -\sqrt{\beta - \pi_C\sigma^2 + \sigma^2} = -\sqrt{\beta - (\pi_C - 1)\sigma^2}.$$

□

B.2 Impact of Racial Competency on Winning Probability

Proposition 3 (Impact of ρ on the Winning Probability for the Uninformed Incumbent)**.** *As ρ increases, the winning probability for the uninformed incumbent increases.*

Proof of Proposition 3. Assume that voters re-elect the incumbent if and only if the state ω falls in the interval $[-\omega^P(\rho), \omega^P(\rho)]$, where

$$\omega^P(\rho) = \sqrt{\beta - \pi_C\sigma^2 + \rho\sigma^2}.$$

Then the uninformed type's winning probability is

$$P^L_{win}(\rho) = \int_{-\omega^P}^{\omega^P} f(\omega)\, d\omega = F(\omega^P(\rho)) - F(-\omega^P(\rho)).$$

To show that $P^L_{win}(\rho)$ is increasing in ρ, we differentiate it with respect to ρ using partial derivatives. By the chain rule,

$$\frac{\partial P^L_{win}(\rho)}{\partial \rho} = \frac{\partial}{\partial \rho}\Big[F(\omega^P(\rho)) - F(-\omega^P(\rho))\Big] = F'(\omega^P(\rho))\frac{\partial \omega^P(\rho)}{\partial \rho} - F'(-\omega^P(\rho))\frac{\partial(-\omega^P(\rho))}{\partial \rho}.$$

Since $F'(x) = f(x)$ and $\frac{\partial(-\omega^P(\rho))}{\partial\rho} = -\frac{\partial \omega^P(\rho)}{\partial \rho}$, this becomes

$$\frac{\partial P^L_{win}(\rho)}{\partial \rho} = f(\omega^P(\rho))\frac{\partial \omega^P(\rho)}{\partial \rho} + f(-\omega^P(\rho))\frac{\partial \omega^P(\rho)}{\partial \rho} = \Big[f(\omega^P(\rho)) + f(-\omega^P(\rho))\Big]\frac{\partial \omega^P(\rho)}{\partial \rho}.$$

Next, differentiate $\omega^P(\rho)$ with respect to ρ. We have

$$\omega^P(\rho) = \Big(\beta - \pi_C\sigma^2 + \rho\sigma^2\Big)^{1/2},$$

so that

$$\frac{\partial \omega^P(\rho)}{\partial \rho} = \frac{1}{2}\Big(\beta - \pi_C\sigma^2 + \rho\sigma^2\Big)^{-1/2} \cdot \sigma^2 = \frac{\sigma^2}{2\omega^P(\rho)}.$$

Substituting back, we obtain

$$\frac{\partial P^L_{win}(\rho)}{\partial \rho} = \Big[f(\omega^P(\rho)) + f(-\omega^P(\rho))\Big]\frac{\sigma^2}{2\omega^P}.$$

Since $\sigma^2 > 0$, $\omega^P > 0$ (by the parameter conditions ensuring the expression under the square root is positive), and f is positive on its support, it follows that

$$\frac{\partial P^L_{win}(\rho)}{\partial \rho} > 0.$$

Thus, $P^L_{win}(\rho)$ is strictly increasing in ρ, which proves that as the probability of a universal second-period domain increases, so does the winning probability for the Uninformed incumbent. □

B.3 Impact of Prior Beliefs on Mixing Probabilities

Proposition 4 (Impact of Prior Beliefs on Mixing Probabilities)**.** *Let the mixing probabilities for the uninformed incumbent be $\Sigma \in (\underline{\Sigma}, \overline{\Sigma})$. Then,*

1. *An increase in the prior belief that the incumbent is high-type raises the probability of posturing right, $\frac{d\overline{\Sigma}}{d\pi_I} > 0$, and lowers the probability of posturing left, $\frac{d\underline{\Sigma}}{d\pi_I} < 0$.*
2. *An increase in the prior belief that the challenger is high-type lowers the probability of posturing right, $\frac{d\overline{\Sigma}}{d\pi_C} < 0$, and raises the probability of posturing left, $\frac{d\underline{\Sigma}}{d\pi_C} > 0$.*

Proof of Proposition 4. Let $x^P(\rho)$ be the equilibrium policy choice by the incumbent. We want to show that the posterior belief

$$\tilde{\pi}_I(x_1^P(\rho)) \;=\; \frac{\pi_I\left[F(\omega^P(\rho)) - F(0)\right]}{\pi_I\left[F(\omega^P(\rho)) - F(0)\right] + (1-\pi_I)\,\Sigma}$$

is strictly increasing in π_I. Define $\omega^P(\rho) = x_1^P(\rho)$. Then

$$\frac{d}{d\pi_I}\left[\tilde{\pi}_I(x_1^P(\rho))\right] \;=\; \underbrace{\frac{\partial\,\tilde{\pi}_I(x_1^P(\rho))}{\partial \pi_I}\Big(\omega^P(\rho)\Big)}_{\text{direct}} + \underbrace{\frac{\partial\,\tilde{\pi}_I(x_1^P(\rho))}{\partial x_1^P(\rho)}\,\frac{d\,x_1^P(\rho)}{d\pi_I}}_{\text{indirect}}.$$

1. Direct effect. Holding $\omega^P(\rho)$ fixed, write

$$\frac{\partial\,\tilde{\pi}_I(x_1^P(\rho))}{\partial \pi_I} =$$

$$\frac{\left(\frac{\partial}{\partial \pi_I}\left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right)\right]\right)\left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right) + (1-\pi_I)\,\Sigma\right] - \left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right)\right]\left(\frac{\partial}{\partial \pi_I}\left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right) + (1-\pi_I)\,\Sigma\right]\right)}{\left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right) + (1-\pi_I)\,\Sigma\right]^2}.$$

Now compute each partial derivative:

$$\frac{\partial}{\partial \pi_I}\left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right)\right] \;=\; F(\omega^P(\rho)) - F(0),$$

$$\frac{\partial}{\partial \pi_I}\left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right) + (1-\pi_I)\,\Sigma\right] \;=\; \left(F(\omega^P(\rho)) - F(0)\right) \;-\; \Sigma.$$

Therefore,

$$\frac{\partial\,\tilde{\pi}_I(x_1^P(\rho))}{\partial \pi_I} \;=\; \frac{\left[F(\omega^P(\rho)) - F(0)\right]\left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right) + (1-\pi_I)\,\Sigma\right] - \left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right)\right]\left[\left(F(\omega^P(\rho)) - F(0)\right) - \Sigma\right]}{\left[\pi_I\left(F(\omega^P(\rho)) - F(0)\right) + (1-\pi_I)\,\Sigma\right]^2}.$$

Break the numerator into two parts:

$$
\begin{aligned}
&\big[F(\omega^P(\rho)) - F(0)\big]\,\big[\pi_I\,(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\,\Sigma\big] \\
&\quad - \,\big[\pi_I\,(F(\omega^P(\rho)) - F(0))\big]\,\big[(F(\omega^P(\rho)) - F(0)) - \Sigma\big] \\
&= \underbrace{\pi_I\,\big(F(\omega^P(\rho)) - F(0)\big)^2 \;+\; (1-\pi_I)\,(F(\omega^P(\rho)) - F(0))\,\Sigma}_{\text{first term}} \\
&\quad - \,\underbrace{\pi_I\,\big(F(\omega^P(\rho)) - F(0)\big)^2 \;-\; \pi_I\,(F(\omega^P(\rho)) - F(0))\,\Sigma}_{\text{second term}} \\
&= \big[\pi_I\,(F(\omega^P(\rho)) - F(0))^2 + (1-\pi_I)(F(\omega^P(\rho)) - F(0))\,\Sigma\big] \\
&\quad - \,\big[\pi_I\,(F(\omega^P(\rho)) - F(0))^2 - \pi_I\,(F(\omega^P(\rho)) - F(0))\,\Sigma\big] \\
&= \big[F(\omega^P(\rho)) - F(0)\big]\,\Sigma.
\end{aligned}
$$

Hence

$$
\frac{\partial\,\tilde{\pi}_I(x_1^P(\rho))}{\partial \pi_I} \;=\; \frac{\big[F(\omega^P(\rho)) - F(0)\big]\,\Sigma}{\Big[\pi_I\,\big(F(\omega^P(\rho)) - F(0)\big) + (1-\pi_I)\,\Sigma\Big]^2}.
$$

2. Indirect effect. First compute $\partial\,\tilde{\pi}_I(x_1^P(\rho))/\partial x_1^P(\rho)$ with $\omega^P(\rho) = x_1^P(\rho)$ held fixed in the denominator:

$$
\frac{\partial\,\tilde{\pi}_I(x_1^P(\rho))}{\partial x_1^P(\rho)} = \frac{\big[\pi_I\,(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\,\Sigma\big]\,\pi_I\,f(\omega^P(\rho) \;-\; \pi_I\,(F(\omega^P(\rho)) - F(0))\,\pi_I\,f(\omega^P(\rho))}{\big[\pi_I\,(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\,\Sigma\big]^2}.
$$

Factor $\pi_I\,f(\omega^P(\rho))$ in the numerator:

$$
\begin{aligned}
&\pi_I\,f(\omega^P(\rho))\,\Big[\big(\pi_I(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\,\Sigma\big) - \pi_I(F(\omega^P(\rho)) - F(0))\Big] \\
&\quad = \;\pi_I\,f(\omega^P(\rho))\,(1-\pi_I)\,\Sigma \;=\; \pi_I(1-\pi_I)\,\Sigma\,f(\omega^P(\rho)).
\end{aligned}
$$

Thus

$$
\frac{\partial\,\tilde{\pi}_I(x_1^P(\rho))}{\partial x_1^P(\rho)} \;=\; \frac{\pi_I(1-\pi_I)\,\Sigma\,f(\omega^P(\rho))}{\Big[\pi_I\,(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\,\Sigma\Big]^2}.
$$

Finally, since $x_1^P(\rho) = \omega^P(\rho)$ does *not* depend on π_I,

$$
\frac{d\,x_1^P(\rho)}{d\pi_I} \;=\; 0,
$$

so the indirect effect is

$$
\frac{\partial\,\tilde{\pi}_I(x_1^P(\rho))}{\partial x_1^P(\rho)} \cdot \frac{d\,x_1^P(\rho)}{d\pi_I} \;=\; \frac{\pi_I(1-\pi_I)\,\Sigma\,f(\omega^P(\rho))}{\Big[\pi_I(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\,\Sigma\Big]^2} \;\cdot\; 0 \;=\; 0.
$$

3. Total derivative. Summing direct and indirect effects,

$$\frac{d}{d\pi_I}\left[\tilde{\pi}_I(x_1^P(\rho))\right] = \underbrace{\frac{(F(\omega^P(\rho)) - F(0))\,\Sigma}{\left[\pi_I(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\,\Sigma\right]^2}}_{\text{direct}} + 0$$

$$= \frac{(F(\omega^P(\rho)) - F(0))\,\Sigma}{\left[\pi_I(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\,\Sigma\right]^2} > 0.$$

Therefore, $\tilde{\pi}_I(x_1^P(\rho))$ is strictly increasing in π_I.

4. To show $\frac{d\overline{\Sigma}}{d\pi_I} > 0$.

Now, suppose $\tilde{\pi}_I(x_1^P(\rho)) = \pi_C$. We established that $\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\pi_I} > 0$. Now, to calculate the impact of π_I on $\overline{\Sigma}$ we use the Implicit Function Theorem which states that

$$\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\pi_I} + \frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\overline{\Sigma}}\frac{d\overline{\Sigma}}{d\pi_I} = 0.$$

First, to determine the sign of $\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\overline{\Sigma}}$, we use the quotient rule:

$$\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\overline{\Sigma}} = \frac{\pi_I(F(\omega^P(\rho)) - F(0))}{\pi_I(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\overline{\Sigma}}$$

$$\iff \; -\frac{1-\pi_I}{[\pi_I(F(\omega^P(\rho)) - F(0)) + (1-\pi_I)\overline{\Sigma}]^2} < 0.$$

Then, subtract $\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\pi_I}$ from the LHS:

$$\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\overline{\Sigma}}\frac{d\overline{\Sigma}}{d\pi_I} = -\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\pi_I}.$$

After, divide $\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\overline{\Sigma}}$ from the LHS:

$$\frac{d\overline{\Sigma}}{d\pi_I} = -\frac{\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\pi_I}}{\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\overline{\Sigma}}}.$$

Because $\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\overline{\Sigma}} < 0$ and $\frac{\partial\tilde{\pi}_I(x_1^P(\rho))}{\pi_I} > 0$, $\frac{d\overline{\Sigma}}{d\pi_I} > 0$. Therefore, increasing the prior belief that an incumbent is high-type increases $\overline{\Sigma}$. An analogous case is made for $\underline{\Sigma}$.

5. To show $\frac{d\overline{\Sigma}}{d\pi_C} < 0$.

First, we solve the state threshold with respect to π_C. Let $\omega^P(\rho) = \sqrt{\beta - \pi_C\sigma^2 + \rho\sigma^2}$. We want to show that

$$\frac{\partial \omega^P(\rho)}{\partial \pi_C} < 0.$$

By the chain rule, let

$$\omega^P(\pi_C) = (f(\pi_C))^{\frac{1}{2}}.$$

Then,

$$\frac{d\omega^P(\rho)}{d\pi_C} = \frac{1}{2}(f(\pi_C))^{-\frac{1}{2}},$$

and

$$\frac{d(\beta - \pi_C\sigma^2 + \rho\sigma^2)}{d\pi_C} = -\sigma^2.$$

Multiplying results in

$$\frac{\partial \omega^P(\rho)}{\partial \pi_C} = -\frac{\sigma^2}{2\omega^P(\rho)} < 0.$$

Suppose $\tilde{\pi}_I(x^P(\rho)) = \pi_C$. Then, simple algebra results in

$$\overline{\Sigma} = (\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}F(\omega^P(\rho)) - F(0)).$$

We want to show that $\frac{\partial \overline{\Sigma}}{\partial \pi_C} < 0$. To do so, we will utilize the multivariate chain rule. First, the direct effect:

$$\begin{aligned} \frac{\partial \overline{\Sigma}}{\partial \pi_C} &= \frac{\pi_I}{1-\pi_I} \cdot \frac{d}{d\pi_C}(\frac{1-\pi_C}{\pi_C}) \\ \iff \quad & \frac{\pi_I}{1-\pi_I} \cdot -\frac{1}{(\pi_C)^2} \cdot F(\omega^P) - F(0) \\ &= -\frac{\pi_I(F(\omega^P(\rho)) - F(0))}{(\pi_C)^2}. \end{aligned}$$

Next, the indirect effect:

$$\frac{\partial \overline{\Sigma}}{\partial \omega^P(\rho)} = \frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)} f(\omega^P(\rho)),$$

and

$$\frac{\partial \omega^P(\rho)}{\partial \pi_C} = -\frac{\sigma^2}{2\omega^P},$$

from above. Then, to calculate the total derivative via the multivariate chain rule:

$$\frac{d\overline{\Sigma}}{d\pi_C} = -\frac{\pi_I(F(\omega^P(\rho)) - F(0))}{(\pi_C)^2} + \frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)} f(\omega^P(\rho)) \cdot -\frac{\sigma^2}{2\omega^P} < 0.$$

The result of the multivariate chain rule is that $\frac{d\overline{\Sigma}}{d\pi_C} < 0$. Therefore, increasing the prior belief that a challenger is high-type decreases $\overline{\Sigma}$. An analogous case is made for $\underline{\Sigma}$. □

C Extensions

C.1 Ideology

Lemma 3. (Competitive Elections). *Suppose the election is competitive. Then,*

1. *There is a perfect Bayesian equilibrium analogous to proposition 1.*
2. *As polarization increases, so does ω_R^P.*

Proof of Lemma 3. The first part of Lemma 3 follows from Proposition 1.

1. Uninformed Incumbent.

Suppose $\pi_I = \pi_C = \pi$. The uninformed type's utility is

$$\beta - \sigma - (R-L)^2 - (1-\pi)\sigma^2,$$

For selecting R. For $\overline{x}_R^P$ (analogous case for $\underline{x}_R^P$,

$$\beta - \sigma^2 - (\overline{x}_R^P - L)^2 - (1-\pi)\sigma^2.$$

Choosing any $x_1 \in (\underline{x}_R^P, \overline{x}_R^P)$ yields a payoff that is weakly worse than choosing $x_1 = R$. Moreover, selecting $x_1 > \overline{x}_R^P$ or $x_1 < \underline{x}_R^P$ is strictly worse than choosing $x_1 = \overline{x}_R^P$ or $x_1 = \underline{x}_R^P$. Therefore, the uninformed incumbent is indifferent between $x_1 > \overline{x}_R^P$, $x_1 < \underline{x}_R^P$, and $x_1 = R$. The uninformed type mix in equilibrium.

2. Informed Incumbent.

For the informed type, if $\omega_1 > \overline{x}_R^P$ or $\omega_1 < \underline{x}_R^P$, the optimal policy is to choose $x_1 = \omega_1 + R$. Now partition the state space such that for $\omega_1 \in [0, \overline{x}_R^P]$, the optimal policy for re-election is $x_1 = \overline{x}_R^P$. If the informed type selected $x_1 = \omega_1 + R$, they would be removed from office. Thus, their payoffs are

$$\beta - (1-\pi)\sigma^2 - (L-R)^2,$$

if removed. And,

$$-(\overline{x}_R^P - \omega_1)^2 + 2\beta,$$

if re-elected. The incumbent will not deviate from $\overline{x}_R^P$ if

$$\begin{aligned}
-(x_R^P)^2 + 2\beta &> \beta - (1-\pi)\sigma^2 - (L-R)^2 \\
-(x_R^P)^2 + \beta &> -(1-\pi)\sigma^2 - (L-R)^2 \\
\beta + (1-\pi)\sigma^2 + (L-R)^2 &> (x_R^P)^2 \\
\sigma^2 - \pi\sigma^2 &> \rho\sigma^2 - \pi\sigma^2 \\
\sigma^2 &> \rho\sigma^2.
\end{aligned}$$

where $(x_R^P)^2 = \beta + \rho\sigma^2 - \pi\sigma^2 + (R-L)^2$. This inequality holds for $\rho \in [0,1)$. A similar argument exists for $\omega_1 \in (\underline{x}_R^P, 0)$. Thus, the $\omega_1 = 0$ type prefers $x_1 = \overline{x}_R^P$ over $x_1 = \omega_1 + R$.

3. Voter Decision.

When $x_1 = \overline{x}_R^P$ is is required that $\tilde{\pi}_I(x_1) \geq \pi_C$ for the incumbent to be re-elected. The voter's expected utility for re-electing the incumbent is

$$-R^2 - (1 - \tilde{\pi}_I(x_1))\sigma^2,$$

and for electing the challenger,

$$-L^2 - (1-\pi_C)\sigma^2.$$

Thus,

$$\begin{aligned}
-R^2 - \big(1 - \tilde{\pi}_I(x_1)\big)\sigma^2 &\geq -L^2 - \big(1-\pi_C\big)\sigma^2 \\
-R^2 - \sigma^2 + \tilde{\pi}_I(x_1)\sigma^2 &\geq -L^2 - \sigma^2 + \pi_C\sigma^2 \\
\tilde{\pi}_I(x_1) &\geq \pi_C + \frac{R^2 - L^2}{\sigma^2},
\end{aligned}$$

where my definition of a competitive election $0 < R^2 - L^2 < 1$. By a slightly revised version of Lemma 2,

$$\tilde{\pi}_I(x_1) = \frac{(F(\omega_R^P) - F(0)) \cdot \pi_I}{(F(\omega_R^P) - F(0)) \cdot \pi_I - (1-\pi_C)\overline{\Sigma}_R} \geq \pi_C + \frac{R^2 - L^2}{\sigma^2}.$$

Let $A := (F(\omega_R^P)$, $B := F(0)$, $\Delta = \frac{R^2-L^2}{\sigma^2}$. Then,

$$\begin{aligned}
\tilde{\pi}_I(x_1) = \frac{(A-B)\cdot\pi}{(A-B)\cdot\pi - (1-\pi)\,\overline{\Sigma}_R} &\geq \pi + \Delta \\
(A-B)\pi &\geq (\pi+\Delta)\big[\pi(A-B) + (1-\pi)\,\overline{\Sigma}_R\big] \\
(A-B)\pi &\geq (\pi+\Delta)(\pi(A-B)) + (\pi+\Delta)(1-\pi)\,\overline{\Sigma}_R \\
(A-B)\pi - \pi(A-B)(\pi+\Delta) &\geq (\pi+\Delta)(1-\pi)\,\overline{\Sigma}_R \\
(A-B)\pi\big[1-(\pi+\Delta)\big] &\geq (\pi+\Delta)(1-\pi)\,\overline{\Sigma}_R \\
\frac{(A-B)\pi\big[1-\pi-\Delta\big]}{(\pi+\Delta)(1-\pi)} &\geq \overline{\Sigma}_R \\
\frac{\pi}{1-\pi}\cdot\frac{1-\pi-\Delta}{\pi+\Delta} &\geq \overline{\Sigma}_R.
\end{aligned}$$

A similar argument holds for $x_1 = \underline{x}_R^P$.

The second part of Lemma 3 comes from differentiating ω_R^P WRT $R - L$.

□

Proposition 5 (Comparative Statics Under Symmetric Polarization)**.** *Suppose the incumbent's and challenger's biases are equidistant* ($R = -L$)*. Then,*

1. ***Effect of polarization.*** *The incumbent's winning probability increases in polarization:*

$$\frac{\partial P_{win}^L}{\partial R} > 0.$$

2. ***Effect of racial competency.*** *The incumbent's winning probability increases in the probability of a universal second period:*

$$\frac{\partial P_{win}^L}{\partial \rho} > 0.$$

3. ***Interaction effect on the state threshold.*** *The marginal benefit of polarization on the state threshold decreases as* ρ *increases:*

$$\frac{\partial \omega_R^P}{\partial R\,\partial \rho} < 0.$$

4. ***Interaction effect on the winning probability.*** *The marginal benefit of polarization on the incumbent's winning probability decreases as ρ increases:*

$$\frac{\partial P^L_{win}}{\partial R\,\partial \rho} < 0.$$

Proof of Proposition 5.
1. To show $\frac{\partial P^L_{win}}{\partial R} > 0$.

Suppose that the politicians are symmetrically polarized such that $R = -L$. Further, define the winning probability for the low-type as

$$P^L_{win} := \pi + (1-\pi)[\overline{\Sigma}_R + \underline{\Sigma}_R].$$

We want to show that $\frac{\partial P^L_{win}(R)}{\partial R} > 0$. First, expand the mixing probability such that

$$\pi + (1-\pi)[\frac{\pi}{1-\pi}\frac{1-\pi-\frac{(R^2-(R^2))}{\sigma^2}}{\pi+\frac{(R^2-(R^2))}{\sigma^2}}](F(\omega^P_R) - F(-\omega^P_R)).$$

The mixing probability simplifies to

$$\pi + (1-\pi)(F(\omega^P_R) - F(-\omega^P_R)).$$

Next, to solve the partial derivative WRT to R, we utilize the chain rule.

$$\frac{\partial P^L_{win}}{\partial R} = (1-\pi)[f(\omega^P_R)\frac{\partial \omega^P_R}{\partial R} - f(-\omega^P_R)\frac{\partial(-\omega^P_R)}{\partial R}].$$

Because $\frac{\partial(-\omega^P_R)}{\partial R} = -1(\frac{\partial(\omega^P_R)}{\partial R})$,

$$\frac{\partial P^L_{win}}{\partial R} = (1-\pi)[f(\omega^P_R)\frac{\partial \omega^P_R}{\partial R} + f(-\omega^P_R)\frac{\partial(\omega^P_R)}{\partial R}].$$

The sign of the winning probability is dependent on $\frac{\partial(\omega^P_R)}{\partial R}$. Let $\omega^P_R = \sqrt{\beta + \rho\sigma^2 - \pi\sigma^2 + (R-L)^2}$. Because of symmetric polarization, $\omega^P_R = \sqrt{\beta + \rho\sigma^2 - \pi\sigma^2 + 4R^2}$. To solve WRT R, we utilize the chain rule:

$$\frac{\partial \omega^P_R}{\partial R} = f'(g(R)) \cdot g'(R),$$

where $g(R) = \beta + \rho\sigma^2 - \pi\sigma^2 + 4R^2$. First,

$$f'(g(R)) = \frac{1}{2}(g(R))^{-\frac{1}{2}}.$$

Second,

$$g'(R) = 8R.$$

Combining and simplifying results in

$$\frac{\partial \omega^P_R}{\partial R} = \frac{1}{2}(g(R))^{-\frac{1}{2}} \cdot 8R = \frac{4R}{\omega^P_R}.$$

Because $\frac{\partial \omega_R^P}{\partial R} > 0$, $\frac{\partial P_{win}^L}{\partial R} > 0$. Therefore, as polarization increases, so does the winning probability for the low-type.

2. To show $\frac{\partial P_{win}^L}{\partial \rho} > 0$.

Suppose that the politicians are symmetrically polarized such that $R = -L$. Further, define the winning probability for the low-type as

$$P_{win}^L := \pi + (1-\pi)[\overline{\Sigma}_R + \underline{\Sigma}_R].$$

We want to show that $\frac{\partial P_{win}^L(R)}{\partial \rho} > 0$. Expand the mixing probability such that

$$\pi + (1-\pi)[\frac{\pi}{1-\pi}\frac{1-\pi-\frac{(R^2-(R^2))}{\sigma^2}}{\pi+\frac{(R^2-(R^2))}{\sigma^2}}](F(\omega_R^P) - F(-\omega_R^P)).$$

The mixing probability simplifies to

$$\pi + (1-\pi)(F(\omega_R^P) - F(-\omega_R^P)).$$

Next, to solve the partial derivative WRT to ρ, we utilize the chain rule.

$$\frac{\partial P_{win}^L}{\partial \rho} = (1-\pi)[f(\omega_R^P)\frac{\partial \omega_R^P}{\partial \rho} - f(-\omega_R^P)\frac{\partial(-\omega_R^P)}{\partial \rho}].$$

Because $\frac{\partial(-\omega_R^P)}{\partial \rho} = -1(\frac{\partial(\omega_R^P)}{\partial \rho})$,

$$\frac{\partial P_{win}^L}{\partial \rho} = (1-\pi)[f(\omega_R^P)\frac{\partial \omega_R^P}{\partial \rho} + f(-\omega_R^P)\frac{\partial(\omega_R^P)}{\partial \rho}].$$

The sign of the winning probability is dependent on $\frac{\partial(\omega_R^P)}{\partial \rho}$. Let $\omega_R^P = \sqrt{\beta + \rho\sigma^2 - \pi\sigma^2 + (R-L)^2}$. Because of symmetric polarization, $\omega_R^P = \sqrt{\beta + \rho\sigma^2 - \pi\sigma^2 + 4R^2}$. To solve WRT ρ, we utilize the chain rule:

$$\frac{\partial \omega_R^P}{\partial \rho} = f'(g(\rho)) \cdot g'(\rho),$$

where $g(\rho) = \beta + \rho\sigma^2 - \pi\sigma^2 + 4R^2$. First,

$$f'(g(\rho)) = \frac{1}{2}(g(\rho))^{-\frac{1}{2}}.$$

Second,

$$g'(\rho) = \sigma^2.$$

Combining and simplifying results in

$$\frac{\partial \omega_R^P}{\partial \rho} = \frac{1}{2}g(\rho)^{-\frac{1}{2}} \cdot \sigma^2 = \frac{\sigma^2}{2\omega_R^P}.$$

Because $\frac{\partial \omega_R^P}{\partial \rho} > 0$, $\frac{\partial P_{win}^L}{\partial \rho} > 0$. Therefore, as the probability that the next period is universal increases, so does the winning probability for the low-type.

3. To show $\frac{\partial \omega_R^P}{\partial R \partial \rho} < 0$.

Given that

$$\frac{\partial \omega_R^P}{\partial \rho} = \frac{\sigma^2}{2\omega_R^P},$$

the cross-partial is

$$\frac{\partial \omega_R^P}{\partial R \partial \rho} = \frac{\partial[\frac{\sigma^2}{2\omega_R^P}]}{\partial R}.$$

I want to show that

$$\frac{\partial \omega_R^P}{\partial R \partial \rho} < 0.$$

By the quotient rule, let $N = \sigma^2$ and $D = 2\omega_R^P$. Then,

$$\begin{aligned}
\frac{\partial \omega_R^P}{\partial R \partial \rho} &= \frac{D \cdot N' - N \cdot D'}{D^2} \\
&= \frac{2\omega_R^P \cdot 0 - \sigma^2 \cdot \frac{\partial D}{\partial R}}{(2\omega_R^P)^2} \\
&= -\frac{2\sigma^2}{4(\omega_R^P)^2} \cdot \frac{\partial \omega_R^P}{\partial R} \\
&= -\frac{\sigma^2}{2(\omega_R^P)^2} \cdot \frac{4R}{\omega} \\
&= -\frac{2\sigma^2 R}{(\omega_R^P)^3}.
\end{aligned}$$

Therefore,

$$\frac{\partial \omega_R^P}{\partial R \partial \rho} < 0,$$

implying that the marginal benefit of widening polarization on the state threshold decreases as ρ increases.

4. To show $\frac{\partial P_{win}^L}{\partial R \partial \rho} < 0$.

Suppose that the politicians are symmetrically polarized such that $R = -L$. Further, define the winning probability for the low-type as

$$P_{win}^L := \pi + (1-\pi)[\overline{\Sigma}_R + \underline{\Sigma}_R].$$

We want to show that $\frac{\partial^2 P_{win}^L(R)}{\partial R \partial \rho} < 0$. Expand the mixing probability such that

$$\pi + (1-\pi)[\frac{\pi}{1-\pi} \frac{1 - \pi - \frac{(R^2 - (R^2))}{\sigma^2}}{\pi + \frac{(R^2-(R^2))}{\sigma^2}}](F(\omega_R^P) - F(-\omega_R^P)).$$

The mixing probability simplifies to

$$\pi + (1-\pi)(F(\omega_R^P) - F(-\omega_R^P)).$$

Next, we know that

$$\frac{\partial P_{win}^L(R)}{\partial \rho} = (1-\pi)[f(\omega_R^P) + f(-\omega_R^P)]\frac{\partial \omega_R^P}{\partial \rho}.$$

Now, to locate $\frac{\partial^2 P_{win}^L(R)}{\partial R \partial \rho}$,

$$\frac{\partial^2 P_{win}^L(R)}{\partial R \partial \rho} = \frac{\partial\big[(1-\pi)[f(\omega_R^P) + f(-\omega_R^P)]\frac{\partial \omega_R^P}{\partial \rho}\big]}{\partial R}.$$

Let $A := (1-\pi)[f(\omega_R^P) + f(-\omega_R^P)]$ and $B := \frac{\partial \omega_R^P}{\partial \rho}$. First,

$$\begin{aligned}\frac{\partial A}{\partial R} &= (1-\pi)[f'(\omega_R^P)\frac{\partial \omega_R^P}{\partial R} + f'(-\omega_R^P)\frac{\partial(-\omega_R^P)}{\partial R}] \\ &= (1-\pi)[f'(\omega_R^P) - f'(-\omega_R^P)]\frac{\partial \omega_R^P}{\partial R}.\end{aligned}$$

Second,

$$\frac{\partial B}{\partial R} = \frac{\partial[\frac{\partial \omega_R^P}{\partial \rho}]}{\partial R} = \frac{\partial^2 \omega_R^P}{\partial R \partial \rho},$$

which is equal to $-\frac{2\sigma^2 R}{(\omega_R^P)^3}$. By the product rule,

$$\frac{\partial^2 P_{win}^L(R)}{\partial R \partial \rho} = \frac{\partial A}{\partial R}\cdot B + A\frac{\partial B}{\partial R}$$

$$= \big[(1-\pi)[f'(\omega_R^P) - f'(-\omega_R^P)]\frac{\partial \omega_R^P}{\partial R}\big]\frac{\partial \omega_R^P}{\partial \rho} + \big[(1-\pi)[f(\omega_R^P) + f(-\omega_R^P)]\big]\frac{\partial^2 \omega_R^P}{\partial R \partial \rho}.$$

Rearrange and simplify,

$$(1-\pi)\big\{[f(\omega_R^P) + f(-\omega_R^P)]\frac{\partial^2 \omega_R^P}{\partial R \partial \rho} + [f'(\omega_R^P) - f'(-\omega_R^P)]\frac{\partial \omega_R^P}{\partial R}\frac{\partial \omega_R^P}{\partial \rho}\big\}.$$

We know that

$$\frac{\partial \omega_R^P}{\partial R} > 0 \qquad \frac{\partial \omega_R^P}{\partial \rho} > 0,$$

and,

$$\frac{\partial^2 \omega_R^P}{\partial R \partial \rho} < 0.$$

Therefore,

$$\frac{\partial^2 P_{win}^L(R)}{\partial R \partial \rho} < 0,$$

implying that the marginal benefit of widening polarization on the winning probability decreases as ρ increases.

□

C.2 Legislative Constraints

C.2.1 When $d_1 = U$.

Lemma 4. (Symmetric Legislative Constraint). *Suppose the legislative constraints are strong* ($\Psi < \overline{x}^P(\rho)$

1. *Assume the incumbent is popular* ($\pi_I > \pi_C$).
   - *Informed Incumbent: If* $\omega \geq 0$ *then the incumbent chooses* $x_1 = \Psi$. *If* $\omega < 0$, *then the incumbent chooses* $x_1 = -\Psi$.
   - *Uninformed Incumbent: The incumbent chooses* $x_1 = \Psi$ *with probability* $1 - F(0)$ *and* $x = -\Psi$ *with probability* $F(0)$.
   - *Voter: The voter always re-elects the incumbent.*

2. *Assume the incumbent is unpopular* ($\pi_I < \pi_C$).
   - *Informed Incumbent: If* $\omega \geq 0$ *then the incumbent chooses* $x_1 = \Psi$. *If* $\omega < 0$, *then the incumbent chooses* $x_1 = -\Psi$.
   - *Uninformed Incumbent: The incumbent chooses* $x_1 = \Psi$ *with probability* $\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}\big(1-F(0)\big)$ *and* $x = -\Psi$ *with probability* $\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}F(0)$.
   - *Voter: The voter reelects the incumbent with probability* $\mu(\Psi) = \frac{\Psi^2+\sigma^2}{\beta-(1-\rho)\sigma^2-V_C(\Psi)}$.

Proof.

1. Suppose $\pi_I > \pi_C$. The updated belief that the incumbent is high-type is

   $$\tilde{\pi}_I(\Psi) = \frac{\pi_I(1-F(0))}{\pi_I(1-F(0)) + (1-\pi_I)(1-F(0))} = \pi_I \geq \pi_C.$$

   A high-type will never deviate from selecting the closest bounds - $\Psi(-\Psi)$ in equilibrium.
   Consider now an uninformed type. Choosing some $x_1 \in (-\Psi, \Psi)$ is off-the-path, yielding a strictly worse payoff. By definition of $\overline{x}$, a low-type is indifferent between selecting $x_1 = \Psi$ and $x_1 = 0$. Moreover, the low-type is indifferent between choosing $-\Psi$ or Ψ as they are symmetric around 0. Therefore, the uninformed type mixes in equilibrium.

2. Suppose $\pi_I < \pi_C$. The updated belief that the incumbent is informed is

   $$\tilde{\pi}_I(\Psi) = \frac{\pi_I(1-F(0))}{\pi_I(1-F(0)) + (1-\pi_I)\tilde{\Sigma}},$$

   where $\tilde{\Sigma}$ is the mixing probability of selecting $x_1 = \Psi$ for the low-type.
   Given that $\pi_I < \pi_C$, the voter always prefers the challenger. As such, the low-type

has an incentive to select $x_1 = 0$. For the uninformed type to mix, the voter must be indifferent. Let $\tilde{\pi}_I(\Psi) = \pi_C$ such that

$$\tilde{\pi}_I(\Psi) = \frac{\pi_I(1 - F(0))}{\pi_I(1 - F(0)) + (1 - \pi_I)\tilde{\Sigma}} = \pi_C.$$

Rearranging and,

$$\tilde{\Sigma} = \frac{\pi_I(1 - \pi_C)}{\pi_C(1 - \pi_I)}\big(1 - F(0)\big),$$

which hold as long as the low-type selects $x_1 = \Psi$ with probability $\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}\big(1 - F(0)\big)$. An analogous case is made for $x_1 = -\Psi$.
All that is left is to determine the voter's mixing probability. Define the voter's continuation value for electing the challenger as

$$\begin{aligned} V_C(\Psi) = \pi_C\Big\{\rho\Big[&\int_{-\infty}^{-\Psi} -(-\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2 + \int_{\Psi}^{\infty} -(\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2\Big] \\ &+ (1-\rho)\Big[\int_{-\infty}^{-\Psi} -(-\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2 + \int_{\Psi}^{\infty} -(\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2\Big]\Big\} \\ &+ (1-\pi_C)\Big\{\rho\Big[\int_{-\infty}^{-\Psi} -(-\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2 + \int_{\Psi}^{\infty} -(\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2\Big] \\ &+ (1-\rho)\Big[\int_{-\infty}^{\infty} -\omega_2^2 f(\omega_2)\, d\omega_2\Big]\Big\}. \end{aligned}$$

For the low-type to mix, they need to be indifferent between selecting $x_1 = 0$ and $x_1 = \Psi$. Therefore,

$$\begin{aligned} \beta + V_C(\Psi) &= \beta - \sigma^2 - \Psi^2 + \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] + (1 - \mu(\Psi))\, V_C(\Psi) \\ \iff \quad & \beta = \beta - \sigma^2 - \Psi^2 + \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] - \mu(\Psi)\, V_C(\Psi) \\ \iff \quad & \beta + \mu(\Psi)\, V_C(\Psi) = \beta - \sigma^2 - \Psi^2 + \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] \\ \iff \quad & \mu(\Psi)\, V_C(\Psi) = -\sigma^2 - \Psi^2 + \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] \\ \iff \quad & \mu(\Psi)\, V_C(\Psi) - \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] = -\sigma^2 - \Psi^2 \\ \iff \quad & -\mu(\Psi)\Big[\beta - (1-\rho)\sigma^2 - V_C(\Psi)\Big] = -\sigma^2 - \Psi^2 \\ \iff \quad & \mu(\Psi) = \frac{\Psi^2 + \sigma^2}{\beta - (1-\rho)\sigma^2 - V_C(\Psi)}. \end{aligned}$$

The uninformed type is indifferent between $-\Psi$ and Ψ because they are equidistant from 0 and $\mu(\Psi) = \mu(-\Psi)$. As before, all informed types, $\omega \in (-\Psi, \Psi)$ and $\mid \omega \mid > \mid \Psi \mid$, select the closest bound in equilibrium.

□

C.2.2 When $d_1 = P$.

Lemma C.2.2. (Symmetric Legislative Constraint when $d_1 = P$). *Suppose the first-period policy domain is particular ($d_1 = P$) and the legislative constraints are strong ($\Psi < \overline{x}$).*

1. *Assume the incumbent is popular ($\pi_I > \pi_C$).*
   - *Informed Incumbent: If $\omega \geq 0$ then the incumbent chooses $x_1 = \Psi$. If $\omega < 0$, then the incumbent chooses $x_1 = -\Psi$.*
   - *Uninformed Incumbent: The incumbent chooses $x_1 = \Psi$ with probability $1 - F(0)$ and $x = -\Psi$ with probability $F(0)$.*
   - *Voter: The voter always re-elects the incumbent.*
2. *Assume the incumbent is unpopular ($\pi_I < \pi_C$).*
   - *Informed Incumbent: If $\omega \geq 0$ then the incumbent chooses $x_1 = \Psi$. If $\omega < 0$, then the incumbent chooses $x_1 = -\Psi$.*
   - *Uninformed Incumbent: The incumbent chooses $x_1 = \Psi$ with probability $\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}\big(1 - F(0)\big)$ and $x = -\Psi$ with probability $\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}F(0)$.*
   - *Voter: The voter reelects the incumbent with probability $\mu(\Psi) = \frac{\Psi^2}{\beta-(1-\rho)\sigma^2-V_C(\Psi)}$.*

Proof.

1. Suppose $d_1 = P$ and $\pi_I > \pi_C$. The updated belief that the incumbent is high-type is

$$\tilde{\pi}_I(\Psi) = \frac{\pi_I(1-F(0))}{\pi_I(1-F(0)) + (1-\pi_I)(1-F(0))} = \pi_I \geq \pi_C.$$

   A high-type will never deviate from selecting the closest bounds - $\Psi(-\Psi)$ in equilibrium.
   Consider now an uninformed type. Choosing some $x_1 \in (-\Psi, \Psi)$ is off-the-path, yielding a strictly worse payoff. By definition of $\overline{x}$, a low-type is indifferent between selecting $x_1 = \Psi$ and $x_1 = 0$. Moreover, the low-type is indifferent between choosing $-\Psi$ or Ψ as they are symmetric around 0. Therefore, the uninformed type mixes in equilibrium.

2. Suppose $d_1 = P$ and $\pi_I < \pi_C$. The updated belief that the incumbent is informed is

$$\tilde{\pi}_I(\Psi) = \frac{\pi_I(1-F(0))}{\pi_I(1-F(0)) + (1-\pi_I)\tilde{\Sigma}},$$

   where $\tilde{\Sigma}$ is the mixing probability of selecting $x_1 = \Psi$ for the low-type.
   Given that $\pi_I < \pi_C$, the voter always prefers the challenger. As such, the low-type

has an incentive to select $x_1 = 0$. For the uninformed type to mix, the voter must be indifferent. Let $\tilde{\pi}_I(\Psi) = \pi_C$ such that

$$\tilde{\pi}_I(\Psi) = \frac{\pi_I(1 - F(0))}{\pi_I(1 - F(0)) + (1 - \pi_I)\tilde{\Sigma}} = \pi_C.$$

Rearranging and,

$$\tilde{\Sigma} = \frac{\pi_I(1 - \pi_C)}{\pi_C(1 - \pi_I)}\big(1 - F(0)\big),$$

which hold as long as the low-type selects $x_1 = \Psi$ with probability $\frac{\pi_I(1-\pi_C)}{\pi_C(1-\pi_I)}\big(1 - F(0)\big)$. An analogous case is made for $x_1 = -\Psi$.

All that is left is to determine the voter's mixing probability. Define the voter's continuation value for electing the challenger as

$$\begin{aligned} V_C(\Psi) = \pi_C\Big\{\rho\Big[\int_{-\infty}^{-\Psi} -(-\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2 + \int_{\Psi}^{\infty} -(\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2\Big] \\ + (1-\rho)\Big[\int_{-\infty}^{-\Psi} -(-\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2 + \int_{\Psi}^{\infty} -(\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2\Big]\Big\} \\ + (1-\pi_C)\Big\{\rho\Big[\int_{-\infty}^{-\Psi} -(-\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2 + \int_{\Psi}^{\infty} -(\Psi - \omega_2)^2 f(\omega_2)\, d\omega_2\Big] \\ + (1-\rho)\Big[\int_{-\infty}^{\infty} -\omega_2^2 f(\omega_2)\, d\omega_2\Big]\Big\}. \end{aligned}$$

For the low-type to mix, they need to be indifferent between selecting $x_1 = 0$ and $x_1 = \Psi$. Therefore,

$$\begin{aligned} \beta - \sigma^2 + V_C(\Psi) &= \beta - \sigma^2 - \Psi^2 + \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] + (1 - \mu(\Psi))\, V_C(\Psi) \\ \iff \quad & \beta - \sigma^2 = \beta - \sigma^2 - \Psi^2 + \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] - \mu(\Psi)\, V_C(\Psi) \\ \iff \quad & \beta - \sigma^2 + \mu(\Psi)\, V_C(\Psi) = \beta - \sigma^2 - \Psi^2 + \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] \\ \iff \quad & \mu(\Psi)\, V_C(\Psi) = -\Psi^2 + \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] \\ \iff \quad & \mu(\Psi)\, V_C(\Psi) - \mu(\Psi)\big[\beta - (1-\rho)\sigma^2\big] = -\Psi^2 \\ \iff \quad & -\mu(\Psi)\Big[\beta - (1-\rho)\sigma^2 - V_C(\Psi)\Big] = -\Psi^2 \\ \iff \quad & \mu(\Psi) = \frac{\Psi^2}{\beta - (1-\rho)\sigma^2 - V_C(\Psi)}. \end{aligned}$$

The uninformed type is indifferent between $-\Psi$ and Ψ because they are equidistant from 0 and $\mu(\Psi) = \mu(-\Psi)$. As before, all informed types - $\omega \in (-\Psi, \Psi)$ and $\mid \omega \mid > \mid \Psi \mid$ - select the closest bound in equilibrium.

□

Proposition 6 (Impact of ρ on Voters' Perceptions)**.** *As ρ increases, the re-election probability for the uninformed incumbent increases.*

Proof of Proposition 6. Define the total derivative as

$$\frac{d\mu(\Psi^U)}{d\rho} = \frac{\partial\mu(\Psi^U)}{\partial\rho} + \left[\frac{\partial\mu(\Psi^U)}{\partial\mathbb{E}_{U_I}(x(\Psi^U))} \cdot \frac{\partial\mathbb{E}_{U_I}(x(\Psi^U))}{\partial\rho} + \frac{\partial\mu(\Psi^U)}{\partial V_C(\Psi^U)} \cdot \frac{\partial V_C(\Psi^U)}{\partial\rho}\right].$$

There is no direct effect, so

$$\frac{\partial\mu(\Psi^U)}{\partial\rho} = 0.$$

To locate the indirect effect:

1. **Indirect effect of $\mathbb{E}_{U_I}(x(\Psi^U))$.** Let

$$D := \beta - (1-\rho)\sigma^2 - V_C(\Psi^U), \qquad N := \Psi^2 + \sigma^2.$$

Then

$$\frac{\partial\mu(\Psi^U)}{\partial\mathbb{E}_{U_I}(x(\Psi^U))} = \frac{D \cdot N' - N \cdot D'}{D^2} = \frac{\Psi^2 + \sigma^2}{\left[\beta - (1-\rho)\sigma^2 - V_C(\Psi^U)\right]^2} > 0.$$

Also,

$$\frac{\partial\mathbb{E}_{U_I}(x(\Psi^U))}{\partial\rho} = -(1-\rho)\sigma^2 = \sigma^2 > 0.$$

2. **Indirect effect of $V_C(\Psi^U)$.** With the same D, N as above,

$$\frac{\partial\mu(\Psi^U)}{\partial V_C(\Psi^U)} = \frac{\Psi^2 + \sigma^2}{\left[\beta - (1-\rho)\sigma^2 - V_C(\Psi^U)\right]^2}.$$

Define

$$A(\Psi^U) := \int_{-\infty}^{-\Psi^U} -(\Psi^U - \omega)^2 f(\omega)\, d\omega + \int_{\Psi^U}^{\infty} -(\Psi^U - \omega)^2 f(\omega)\, d\omega,$$

and

$$B := \int_{-\infty}^{\infty} -\omega^2 f(\omega)\, d\omega = -\sigma^2.$$

Then

$$V_C(\Psi^U) = \pi_C\left[\rho A(\Psi^U) + (1-\rho)A(\Psi^U)\right] + (1-\pi_C)\left[\rho A(\Psi^U) + (1-\rho)B\right].$$

Thus

$$\frac{dV_C(\Psi^U)}{d\rho} = \pi_C\left[A(\Psi^U) - A(\Psi^U)\right] + (1-\pi_C)\left[A(\Psi^U) - B\right] = (1-\pi_C)\left[A(\Psi^U) - B\right].$$

By change of variables,

$$A(\Psi^U) = 2\int_{\Psi^U}^{\infty} -(\omega - \Psi^U)^2 f(\omega)\, d\omega,$$

so

$$\frac{dV_C(\Psi^U)}{d\rho} = (1-\pi_C)\left[2\int_{\Psi^U}^{\infty} -(\omega - \Psi^U)^2 f(\omega)\, d\omega - (-\sigma^2)\right] > 0.$$

3. **Total derivative.** Substituting the components,

$$\frac{d\mu(\Psi^U)}{d\rho} = \frac{\Psi^2 + \sigma^2}{\left[\beta - (1-\rho)\sigma^2 - V_C(\Psi^U)\right]^2} \left\{ \sigma^2 + (1 - \pi_C) \left[2 \int_{\Psi^U}^{\infty} -(\omega - \Psi^U)^2 f(\omega)\, d\omega + \sigma^2 \right] \right\}$$
$$> 0.$$

□